Granada (Spain)
June 24 - 28, 2024

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IMAG CONFERENCE ON ORTHOGONAL POLYNOMIALS, SPECIAL FUNCTIONS AND APPLICATIONS - OPSFA17

GRANADA (SPAIN) JUNE 24 - 28, 2024

DEDICATED TO ANDRÉ RONVEAUX (1932-2023) AND PASCAL MARONI (1933-2024)

Contents

Posters

Invited Talk

Monday 24
9:30
Aula Magna

Sharing orthogonal polynomials with Pascal Maroni and André Ronveaux

Francisco Marcellán

Abstract

In this presentation we summarize the main and innovative contributions by Pascal Maroni and André Ronveaux in the theory of orthogonal polynomials. They were involved in the milestones of the theory of semiclassical and Laguerre-Hahn linear functionals, among others. Basically they focused the attention on the analysis of their algebraic and structural properties from the distributional equations satisfied by them as well as the differential equations of the corresponding Stieltjes functions of the linear and Riccati type, respectively (see [1], [5]). They also wrote joint papers on these topics. The contents of some of them will be described ([3]). I also point out the contributions by Pascal in the framework of d-orthogonality [4], an illustrative example of multiple orthogonality as well as those by André concerning Sobolev orthogonality [2].

Their role in the conformation of the international community of orthogonal polynomials and special functions will be emphasized. In particular the scientific cooperation with several countries in Africa and Europe shows their commitment with young researchers.

Keywords: Linear functionals, Orthogonal Polynomials, Semiclassical linear functionals, Laguerre-Hahn linear functionals, Spectral pertubations. d-orthogonality. Sobolev orthogonality.
AMS Classification: 42C05.

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Research supported by project [PID2021- 122154NB-I00], *Ortogonalidad y Aproximación con Aplicaciones en Machine Learning y Teoría de la Probabilidad* funded by MICIU/AEI/10.13039/501100011033 and by "ERDF A Way of making Europe"

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Plenary Talks

Fourier Transforms of Multivariate Orthogonal Polynomials

Monday 24
10:30
Aula Magna

Rabia Aktaş Karaman

Abstract

Some systems of univariate orthogonal polynomials are mapped onto each other by the Fourier transform. The most-studied example is related to the Hermite functions, which are eigenfunctions of the Fourier transform [6]. Previous research showed that the Jacobi and continuous Hahn polynomials can be mapped onto each other in a similar manner [4]. Recently, in the univariate case, the Fourier transforms of the generalized ultraspherical polynomials, the generalized Hermite polynomials, the finite classical orthogonal polynomials and symmetric sequences of finite orthogonal polynomials have been studied and some families of orthogonal functions have been obtained by using these Fourier transforms and the Parseval's identity [5, 7, 8].

In this work, for the multivariate case, by using the Fourier transform and Parseval's identity, some examples of orthogonal systems of this type are introduced and orthogonality relations are discussed. We, first, apply this method for two-variable orthogonal polynomials, then we investigate the Fourier transforms of multivariate orthogonal polynomials on the simplex and on the unit ball [1, 2, 3].

Keywords: Multivariate Orthogonal Polynomials, Fourier Transform, Parseval's Identity, Hypergeometric Function
AMS Classification: 33C50, 33C70, 33C45, 42B10

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Padé approximants and orthogonality beyond the sphere

Thursday 27
9:00
Aula Magna

Marco Bertola

Abstract

Padé approximants are rational expressions approximating a target function. They can be thought of as meromorphic functions on the Riemann sphere. Similarly, the related orthogonal polynomials can be viewed as meromorphic functions with only a single pole at infinity.

What happens if we replace "Riemann sphere" by "Riemann surface of higher genus"? For example if we replace the sphere with the torus, i.e. an elliptic algebraic curve. This point of view opens a window on a new, unexplored landscape.

The next questions, to which I will offer some answers are then:

- How should we define the notion of Padé approximation on a Riemann surface?
- What, if any, relation the new notion bears with a generalization of orthogonal polynomials?
- Can we address questions of asymptotic analysis for large degrees, and can we export existing methods?
- What kind of potential-theoretic aspects are involved?

We will try to explain how to address these questions and how the answers require some cross-pollination between the two areas of approximation theory and algebraic geometry.

Keywords: Orthogonal Polynomials, Approximation Theory, Applications, Potential theory.
AMS Classification: 42C05, 31C12, 14H60.

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The birth of orthogonal polynomials

Claude Brezinski

Wednesday 26
10:00
Aula Magna

Abstract

This talk is based on a joint book in preparation with Michela Redivo-Zaglia
A Chronological History of the Birth and Early Developments of Orthogonal Polynomials.

It is not exaggeration to say that orthogonal polynomials were first introduced and studied on the occasion of topographic and mathematical works on the measurement of the length of the meridian and the attraction of spheroids around 1783 by Adrien Marie Legendre (1752-1833) et Pierre Simon Laplace(1749-1827).

The second and independent starting point of the study of orthogonal polynomials occurred about forty years later from methods for the approximate computation of definite integrals by Carl Friedrich Gauss (1777-1855) in 1814.

In this talk, we analyze the first contributions of these scholars and describe the birth of orthogonal polynomials.

Keywords: Orthogonal Polynomials, History, Legendre, Laplace, Gauss
AMS Classification: 01A55, 33C45, 42C05.

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On orthogonality, rational approximation, quadrature and exponential analysis, in one and more variables

Tuesday 25
9:00
Aula Magna

Annie Cuyt

Abstract

We establish connections between the concept of orthogonal polynomials and several numerical techniques from rational approximation, Gaussian quadrature and exponential analysis, both in one as well as in several variables.

1. Univariate case

For a linear functional $L : \mathbb{C}[t] \to \mathbb{C} : t^i \to e_i$, a sequence of orthogonal polynomials $\{V_m(z)\}_{m\in\mathbb{N}}$ can be defined by

$$L\left(t^i V_m(t)\right) = 0, \qquad i = 0, \dots, m-1.$$

In [7] these formally orthogonal polynomials are called Hadamard polynomials.

With the $V_m(z)$ we can define associated polynomials

$$W_{m-1}(z) = L\left(\frac{V_m(z) - V_m(t)}{z - t}\right)$$

and reverse polynomials

$$\tilde{V}_m(z) = z^m V_m(1/z).$$

The Padé approximant $[m-1/m]_F$ of degree $m-1$ in the numerator and degree m in the denominator to the formal power series

$$F(z) = \sum_{i=0}^{\infty} e_i z^i$$

precisely equals $\tilde{W}_{m-1}(z)/\tilde{V}_m(z)$. Hence the denominator of this Padé approximant is closely related to the orthogonal polynomial $V_m(z)$.

When the e_i are so-called moments, for instance

$$e_i = \int_{-1}^{1} w(t) t^i \, dt, \qquad 0 < \int_{-1}^{1} w(t) \, dt,$$

then

$$F(z) = \int_{-1}^{1} w(t) \frac{1}{1 - tz} \, dt$$

and the m-point Gaussian quadrature rule

$$\int_{-1}^{1} w(t) \frac{1}{1 - tz} \, dt \approx \sum_{i=1}^{m} A_i^{(m)} \frac{1}{1 - z_i^{(m)} z},$$

Insert here, if there exist, the organizations that support your research.

approximating $F(z)$, equals the $[m-1/m]_F$ Padé approximant. The Gaussian nodes $z_i^{(m)}$ are the zeroes of $V_m(z)$ and the weights $A_i^{(m)}$ are given by

$$A_i^{(m)} = \frac{W_{m-1}(z_i^{(m)})}{V_m'(z_i^{(m)})}, \qquad i = 1, \ldots, m.$$

Since this Gaussian quadrature rule exactly integrates polynomials of degree $2m-1$, we also have

$$e_j = \sum_{i=1}^{m} A_i^{(m)} \left(z_i^{(m)}\right)^j, \qquad j = 0, \ldots, 2m-1.$$

Hence the nodes and weights can be obtained as the solution of the exponential analysis or Prony problem [6]

$$e_j = \sum_{i=1}^{m} A_i^{(m)} \exp(j\phi_i^{(m)}), \qquad z_i^{(m)} = \exp(\phi_i^{(m)}), \quad j = 0, \ldots, 2m-1,$$

where only m and the e_j are given. The $z_i^{(m)}$ are the generalized eigenvalues of a Hankel structured generalized eigenvalue problem and the $A_i^{(m)}$ are the solution of a Vandermonde structured linear system [8].

2. Multivariate case

The concept of the formally orthogonal polynomial $V_m(z)$ is generalized in [4], for different radial weight functions, to so-called spherical orthogonal polynomials. The latter differ from several other definitions of multivariate orthogonal polynomials, in that they preserve the connections laid out above.

Homogeneous multivariate Padé approximants, as defined in [2, 3], can also be obtained from the spherical orthogonal polynomials in a similar way as described here. The homogeneous definition satisfies a very strong projection property, in the sense that this multivariate Padé approximant reduces to the univariate Padé approximant on every one-dimensional subspace.

A whole lot of Gaussian cubature rules on the disk can be united in a single approach [1] when developing the existing rules from these spherical orthogonal polynomials. What's more, the nodes and weights of such Gaussian cubature rules on the disk can be obtained as the solution of a multivariate Prony-like system of interpolation conditions [1]. And this brings us to the next connection.

Prony's result that an m-term exponential analysis problem can be solved uniquely from $2m$ samples e_i, is a one-dimensional result. In [5] this result is proven, more than 200 years later, to hold for higher dimensions $d > 1$: a multivariate linear combination of m exponential terms with unknown inner product exponents can, under mild conditions, be fitted using only $(d+1)m$ data.

Keywords: Orthogonal polynomials, Rational approximation, Quadrature rules, Exponential analysis
AMS Classification: 42C05, 41A21, 65D32, 65T40.

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Universality limits via canonical systems

Benjamin Eichinger

Friday 28
10:00
Aula Magna

Abstract

It is often expected that the local statistical behavior of eigenvalues of some system depends only on its local properties; for instance, the local distribution of zeros of orthogonal polynomials should depend only on the local properties of the measure of orthogonality. The most commonly studied case is known as bulk universality, where Christoffel-Darboux kernels have a double scaling limit given by the sine kernel. In this talk, I will discuss the first completely local sufficient condition for bulk universality and, much more generally, necessary and sufficient conditions for regularly varying universality limits. The proofs of these results rely on the de Branges theory of canonical systems, and the results also apply to other self-adjoint systems with 2×2 transfer matrices such as Schrödinger operators.
This talk is based on joint works with Milivoje Lukić, Brian Simanek and Harald Woracek.

Keywords: Orthogonal Polynomials, Universality Limits, Canonical Systems
AMS Classification: 42C05, 47B36, 47B32.

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Friday 28
9:00
Aula Magna

A role of orthogonal polynomials on the equidistribution of points on Manifolds

Ujué Etayo

Abstract

Given a differentiable manifold, one can ask the question of how to distribute a fixed number of points, say N, so that they are "as far away from each other as possible." There are different definitions that include the concept "as far away from each other as possible". Two recurring definitions are: either those points are solutions of a packing problem (or max min type problem) or those points are minimizers of a certain repulsive energy between points (one can think, for example, of the electrostatic potential). The problem of finding the minimizing points or solving the packing problem appears insurmountable difficult even in its simplest cases. As an example, the open problem of giving a set of 7 points on the 2-sphere that minimize the logarithmic potential.

In the case of energy minimizers, a recurring strategy consists of producing sets of points of the manifold randomly (following a very precise probability distribution) whose associated energy is sufficiently close of the minimum value of this energy. This is the spirit behind problem number 7 on S. Smale's list of millennium problems.

Searching for appropriate probability distributions for different repulsive potentials we enter the field of random matrices, determinantal point processes and orthogonal polynomials. Throughout this talk we will present the relationships of these last three objects with the problem of minimizing potentials taking into account the latest advances in the area.

Keywords: Orthogonal Polynomials, equidistributed points, determinantal point processes, random matrix theory.
AMS Classification: 31C12, 31C20, 52A40.

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Wednesday 26
9:00
Aula Magna

Hypergeometric Polynomials with Free Probability Tools

Andrei Martínez-Finkelshtein

Abstract

Mathematics is a highly interconnected field, and ideas that were initially developed in one context can sometimes find unexpectedly fruitful applications in seemingly unrelated domains. A striking example of this is the recent application of tools from free probability theory to the study of the zeros of polynomials.

One such concept is the finite free convolution of polynomials, introduced relatively recently. This concept becomes particularly appealing when applied to hypergeometric polynomials. Remarkably, these polynomials can be represented as a finite free convolution of more elementary building blocks. This representation, combined with the preservation of real zeros and interlacing properties through free convolutions, provides an effective tool for analyzing when all roots of a particular hypergeometric polynomial are real and when they exhibit monotonicity with respect to parameters. Consequently, this approach offers a fresh perspective on the zero properties of hypergeometric polynomials.

Furthermore, this representation remains valid even in the asymptotic regime, allowing us to express the limit zero distribution of generalized hypergeometric polynomials as a free convolution of more "elementary" measures. This convolution can be expressed analytically by combining some integral transforms of these measures, and it turns out that in the case of hypergeometric polynomials, some of these transforms take a particularly simple form.

These results are demonstrated through applications to some families of multiple (or Hermite-Padé) orthogonal polynomials that can be expressed in terms of generalized hypergeometric functions.

This is a joint work with R. Morales (Baylor University) and Daniel Perales (Texas A&M University).

Keywords: Hypergeometric polynomials; Finite free convolution; Free probability; Multiple ortogonal polynomials; Zero asymptotic
AMS Classification: 33C45, 33C20, 42C05, 46L54

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Thursday 27
10:00
Aula Magna

Matrix Bochner Problem and Darboux transformations

Inés Pacharoni

Abstract

Matrix orthogonal polynomials are sequences of matrix-valued polynomials that are pairwise orthogonal with respect to a matrix-valued inner product defined by an $N \times N$ weight matrix $W(x)$. The problem of finding weight matrices $W(x)$ of size $N \times N$ such that any associated sequence of matrix orthogonal polynomials are eigenfunctions of a second-order matrix differential operator is known as the Matrix Bochner Problem. In the scalar case, S. Bochner (1929) proved that, up to an affine change of coordinates, the only weights on the real line satisfying these properties are the classical weights of Hermite, Laguerre, and Jacobi.

The Matrix Bochner Problem is equivalent to the existence of a second-order operator in the algebra $\mathcal{D}(W)$, which consists of all differential operators having a sequence of matrix-valued orthogonal polynomials with respect to W as eigenfunctions. In [1], the authors prove that, under certain assumptions on the algebra $\mathcal{D}(W)$, solutions to the Matrix Bochner Problem can be obtained through a Darboux transformation of a direct sum of classical scalar weights.

The focus of this talk is to explore the relationship between the algebras $\mathcal{D}(W)$ and $\mathcal{D}(\widetilde{W})$ when $\widetilde{W}$ is a Darboux transformation of W. In particular, we are interesed in the case when $\widetilde{W}$ is a direct sum of classical weights. We will also showcase solutions to the Matrix Bochner Problem that cannot be obtained as Darboux transformations of classical scalar weights.

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Asymptotics and zeros of orthogonal polynomials in the plane

Tuesday 25
10:00
Aula Magna

Aron Wennman

Abstract

In this talk I plan to describe some old and new results concerning orthogonal polynomials with respect to (weighted) area measure on Jordan domains and on the plane. The study of their large-degree asymptotics is a classical topic pioneered by Carleman in the 1920's, and these questions have recently received increased attention due to connections to 2D Coulomb gas models.

After outlining the Coulomb gas connection, I will focus on two recent approaches. The first is PDE-based and shares some resemblance with the Riemann-Hilbert approach to orthogonal polynomials on the real line, and is suited to the setting of varying exponential weights [2, 3]. The second approach is adapted to the unweighted setting on a Jordan domain, and is loosely speaking based on extracting information about the orthogonal polynomials from the associated Bergman kernel. Among the main results is a new puzzle piece in the description of a curious dichotomy exhibited by the zeros of orthogonal polynomials for Jordan domains with corners [2], going back to observations by Eiermann and Stahl.

The talk is based on joint works with Erwin Miña Díaz (University of Mississippi) and with Haakan Hedenmalm (KTH).

Keywords: Orthogonal Polynomials, Bergman kernels, Random Matrices
AMS Classification: 42C05, 30C10, 46E22.

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Contributed Talks

$d-$Orthogonal polynomials of Brenke type $(m+1)-$fold symmetric as special case

Tuesday 25
16:30
Room C

Gahami Abdelhamid, Chaggara Hamza

Abstract

The concept of d-orthogonality polynomials, where d is a positive integer, was originally introduced by Van Iseghem and Maroni [4, 5]. They defined it as a linear recurrence relation of higher order satisfied by a polynomial sequence $\{P_n\}_{n\geq 0}$. This notion represents a natural extension of ordinary orthogonality (when d = 1).

The polynomials sequence of Brenke type $\{P_n\}_{n\geq 0}$ are defined by their generating function

$$\sum_{n=0}^{\infty} \frac{P_n(x)}{n!} t^n = A(t)B(xt)$$

where $A(t) = \sum_{n=0}^{\infty} a_n t^n$ and $B(t) = \sum_{n=0}^{\infty} b_n t^n$ such that for all integer n, $a_0 b_n \neq 0$

Numerous studies have focused on this class of polynomials. Notably, T.S. Chihara [2] tackled the first characterization problem, determining all orthogonal polynomial sequences within this class. The second problem involves identifying all (d+1)-fold symmetric, d-orthogonal polynomials of Brenke type, which was resolved by Ben Cheikh and Ben Romdhane in [3]. Recently, in [1] , we provided a characterization of all d-orthogonal polynomials of Brenke type. This work has yielded several new and previously known results. In our presentation, we intend to delve deeper into our prior findings, offering additional details and properties of the newly obtained family, including specific considerations for $(m+1)-$fold symmetric cases.

Keywords: Brenke polynomials, polynomials sequence, d-orthogonal polynomials, $(m+1)-$fold symmetric,

AMS Classification: 33CXX, 33DXX.

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On the construction of two orthogonal families

Monday 24
12:00
Room C

Jorge Arvesú

Abstract

We study two orthogonal families related to two different systems of multiple orthogonal polynomials derived from an AT-system of vector measures, with components supported on a compact set of the real line, and from an Angelesco system of measures supported on a legged star-like set of the complex plane, respectively. Some applications to physics and number theory will be discussed.

Keywords: Orthogonal Polynomials, Approximation Theory, Multiple Orthogonal Polynomials, Angelesco Polynomials, Recurrence Relations
AMS Classification: 33C45, 33C47, 41A28, 42C05

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Monday 24
15:30
Room B

Rational approximation of Markov functions

Bernhard Beckermann

Abstract

We give some refined L^∞ error estimates for rational approximation of Markov functions, such as $f(z) = 1/\sqrt{z}$ discussed by Zolotarev. In particular, we give a worst case measure, allowing to bound above the relative error on the real axis (and the unit circle) for any other Markov function. Within the class of measures satisfying the Szegő condition, it is known that, asymptotically, the worst case measure with support in $[a, b]$ is the equilibrium measure. This is no longer true in the case of general measures supported on the real line, where we explicitly give for each degree a worst case measure, and compare with the error estimate given by Zolotarev.

Keywords: Rational approximation, Orthogonal Polynomials with varying weights. *AMS Classification:* 33CXX, 42CXX.

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Some families of Bernstein functions

Christian Berg

Wednesday 26
11:30
Room A

Abstract

The set $\mathcal{B}F$ of Bernstein functions plays an important role in probability theory and potential theory, see [2]. It is the set of C^∞-functions $f : (0,\infty) \to [0,\infty)$ satisfying

$$(-1)^{n-1} f^{(n)}(x) \geq 0, \quad x > 0,\, n = 1, 2, \ldots,$$

or equivalently

$$f(x) = a + bx + \int_0^\infty (1 - e^{-xt})\, d\nu(t), \quad x > 0,$$

where $a, b \geq 0$ and the Lévy measure ν is a positive measure on $(0,\infty)$ such that $\int t/(t+1)\, d\nu(t) < \infty$.

We shall mention the important subclasses $\mathcal{C}BF$ and $\mathcal{H}BF$ of $\mathcal{B}F$ consisting respectively of complete Bernstein functions and Horn-Bernstein functions.

The family of functions for $r > 0$

$$f_r(x) = \frac{\log(1 + rx)}{\log(1 + x)}, \quad x > 0$$

has come up in investigations by David Bradley of fractal dimension of Pascal's pyramid modulo a prime. He conjectured that they are Bernstein functions for $0 \leq r \leq 1$. Using complex analysis we have established the stronger statement, that they are complete Bernstein functions for $0 \leq r \leq 1$, while they are Stieltjes functions when $r \geq 1$, see [1].

Keywords: Bernstein functions, Stieltjes functions, Nevanlinna Pick functions
AMS Classification: 30E20, 26A48.

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Thursday 27
12:00
Room B

Markov chains in the theory of multiple orthogonal polynomials

AMÍLCAR BRANQUINHO, JUAN DÍAZ, ANA FOULQUIÉ MORENO, AND MANUEL MAÑAS

Abstract

Hessenberg bounded matrices, which represents non normal operators, of oscilatory type that admit a positive bidiagonal factorization are considered. To motivate the relevance of the oscillatory character the Favard theorem for Jacobi matrices is revisited and it is shown that after an adequate shift of the Jacobi matrix one gets an oscillatory matrix (cf. [2]).

Given a family of orthogonal polynomials or multiple orthogonal polynomials, with a non-negative recurrence matrix, using the Perron–Frobenius theorem, we construct an infinite number of finite Markov chains (cf. [1] and [2]). The hypergeometric expressions for the families of orthogonal polynomials in the Askey diagram, with non-negative recurrence matrices, allow the explicit construction of a diversity of examples of such Markov chains.

Keywords: Multiple orthogonal polynomials, hypergeometric series, Hessenberg matrices, recursion matrix, recurrence, Hahn, Laguerre, Meixner, Jacobi-Piñeiro, AT systems
AMS Classification: 42C05, 33C45, 33C47, 47B39, 47B36.

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Tuesday 25
15:30
Room A

Darboux for CMV, Sobolev and Matrix Orthogonal Polynomials

M. J. Cantero, L. Moral, L. Velázquez

Abstract

Considering a Laurent polynomial modification of a unitary CMV matrix, its Darboux transformation provides another unitary matrix.This transformation is equivalent to a Christoffel modification of the corresponding orthogonality measure.

However, the inverse Darboux transformation does not always provide a new CMV matrix, but leads to the so-called spurious solutions related with discrete Sobolev-type inner products.

In this talk we will show how a connection between discrete Sobolev-type orthogonal polynomials on the unit circle and matrix orthogonal polynomials on the real line allows us to translate a Darboux problem on the unit circle to a matrix Geronimus problem on the real line.

Keywords: Orthogonal Polynomials, Darboux transformation, Sobolev-type inner product.
AMS Classification: 33C45, 42C05.

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Tuesday 25
16:30
Room A

Matrix-valued Discrete-continuous Prolate Operators

William Casper

Abstract

We establish an intrinsic relationship between matrix-valued discrete-continuous bispectral functions and the prolate spheroidal phenomenon. The former functions form a vast class, parametrized by an infinite dimensional manifold, and are constructed by noncommutative Darboux transformations from classical bispectral functions associated to orthogonal polynomials. We prove that all such Darboux transformations which are self-adjoint in a certain sense give rise to integral operators possessing commuting differential operators and to discrete integral operators possessing commuting shift operators. One particularly striking implication of this is the correspondence between discrete and continuous pairs of commuting operators. Moreover, all results are proved in the setting of matrix valued functions, which provides further advantages for applications.

Keywords: Prolate spheroidal functions, discrete-continuous bispectrality, matrix valued bispectral functions, classical orthogonal polynomials
AMS Classification: 37K35, 16S32, 39A70

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Thursday 27
12:00
Room A

Time-and-band limiting for exceptional orthogonal polynomials

Mirta M. Castro Smirnova, F. A. Grünbaum and I. Zurrián

Abstract

The "time-and-band limiting" commutative property was found and exploited by D. Slepian, H. Landau and H. Pollak at Bell Labs in the 1960's, and independently by M. Mehta and later by C. Tracy and H. Widom in Random matrix theory. The property in question is the existence of local operators with simple spectrum that commute with naturally appearing global ones.

Here we give a general result that insures the existence of a commuting differential operator for a given family of exceptional orthogonal polynomials satisfying the "bispectral property". As a main tool we go beyond bispectrality and make use of the notion of Fourier Algebras associated to the given sequence of exceptional polynomials. We illustrate this result with two examples, of Hermite and Laguerre type, exhibiting also a nice Perline's form for the commuting differential operator.

Keywords: Time-band limiting, exceptional polynomials
AMS Classification: 33C45, 22E45, 33C47.

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Positivity of Hankel Transforms

Yong-Kum Cho, Seok-Young Chung and Young Woong Park

Monday 24
17:00
Room B

Abstract

In consideration of the integral transform whose kernel arises as an oscillatory solution of certain second-order linear differential equation, its positivity is investigated on the basis of Sturm's theory. As applications, positivity criteria are obtained for Hankel transforms as well as trigonometric integrals defined on the positive real line.

Keywords: Bessel functions, Fourier transforms, Hankel transforms, oscillatory, positivity, Sturm's oscillation theory.
AMS Classification: 34C10; 42A38; 44A20; 33C10.

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Tuesday 25
15:30
Room B

Chebyshev polynomials and Widom factors

Jacob S. Christiansen, Benjamin Eichinger and Olof Rubin

Abstract

Let $\mathsf{E} \subset \mathbb{C}$ be an infinite compact set, and denote by T_n the minimax (or Chebyshev) polynomials of E, i.e., the monic degree n polynomials minimizing the sup-norm on E. A well-known result by Szegő asserts that $\|T_n\|_{\mathsf{E}} \geq \operatorname{Cap}(\mathsf{E})^n$ for all n, a lower bound that doubles when $\mathsf{E} \subset \mathbb{R}$, as proven by Schiefermayr. More recently, Totik proved that for real subsets, $\|T_n\|_{\mathsf{E}} / \operatorname{Cap}(\mathsf{E})^n \to 2$ if and only if E is an interval. We will introduce the Widom factors, denoted by

$$W_n(\mathsf{E}) := \frac{\|T_n\|_{\mathsf{E}}}{\operatorname{Cap}(\mathsf{E})^n}$$

and investigate whether there exist additional subsets of $\mathbb{C}$ for which $W_n(\mathsf{E}) \to 2$. It appears that the answer is affirmative for certain polynomial preimages. Interestingly, our proof relies on properties of the Jacobi orthogonal polynomials established by Bernstein. We will also discuss the symmetry properties underlying this phenomenon and explore related open problems.

Keywords: Chebyshev Polynomials, Widom Factors, Polynomial Preimages
AMS Classification: 41A50, 30C10, 26D05

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Special functions in discrete Harmonic Analysis

Monday 24
12:00
Room B

Óscar Ciaurri

Abstract

In this talk we want to show the central role played by modified Bessel functions of the first kind in the harmonic analysis related to a multidimensional discrete Laplacian. We will present some properties of these functions and their application in the analysis of classical operators in harmonic analysis. The results in this talk can be seen in [1].

Keywords: Bessel functions, discrete Harmonic Analysis
AMS Classification: 42C10.

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AEI

Thursday 27
15:30
Room B

Sobolev orthogonal polynomials: Connection formulas and zeros

Roberto S. Costas-Santos, Anier Soria-Lorente

Abstract

This contribution aims to obtain several connection formulae for the polynomial sequence, which is orthogonal with respect to the discrete Sobolev inner product

$$\langle f, g\rangle_n = \langle \mathbf{u}, fg\rangle + \sum_{j=1}^{M} \mu_j f^{(\nu_j)}(c_j) g^{(\nu_j)}(c_j), \tag{1}$$

where $\mathbf{u}$ is a classical linear functional, $c_j \in \mathbb{R}$, $\nu_j \in \mathbb{N}_0$, $j = 1, 2, ..., M$.

We later consider the $M = 2$ case, we take the linear Krawtchouk functional $\mathbf{u}^{\mathsf{K}}$, and assume that we have two mass points that can be either outside or inside the convex hull of the support of $\mathbf{u}^{\mathsf{K}}$, and briefly study the behavior of the zeros of the Krawtchouk-Sobolev polynomials, which are orthogonal with respect to the said inner product, in different situations.

Keywords: Fourier coefficients, Sobolev type orthogonal polynomials, Connection formula, Zeros.
AMS Classification: 33C47, 42C05.

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Orthogonal Laurent polynomials of two real variables

Monday 24
12:30
Room A

Ruymán Cruz-Barroso and Lidia Fernández

Abstract

In this talk we consider an appropriate ordering for the Laurent monomials $x^i y^j$, $i, j \in \mathbb{Z}$ that allows us to study sequences of orthogonal Laurent polynomials of the real variables x and y with respect to a positive Borel measure μ defined on $\mathbb{R}^2$ such that $\{x = 0\} \cup \{y = 0\} \not\in \mathrm{supp}(\mu)$. This ordering is suitable for considering the *multiplication plus inverse multiplication operator* on each varibale $\left(x + \frac{1}{x} \text{ and } y + \frac{1}{y}\right)$, and as a result we obtain five-diagonal recurrence relations, Christoffel-Darboux formulas for the reproducing kernel and related Favard's theorems.

Keywords: Orthogonal Laurent polynomials of two real variables, balanced ordering, recurrence relations, Christoffel-Darboux formula and Favard's theorem

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Monday 24
16:30
Room B

Asymptotics and total integrals of the P_I^2 tritronquée solution and its Hamiltonian

Dan Dai and Wen-Gao Long

Abstract

We study the tritronquée solution $u(x,t)$ of the P_I^2 equation, the second member of the Painlevé I hierarchy. This particular solution is also known as the Gurevich-Pitaevskii solution of the KdV equation. It is pole-free on the real line and has various applications in mathematical physics. We obtain a full asymptotic expansion of $u(x,t)$ as $x \to \pm\infty$, uniformly for the parameter t in a large interval. Based on this result, we successfully derive the total integrals of $u(x,t)$ and the associated Hamiltonian with respect to $x \in \mathbb{R}$. Surprisingly, although $u(x,t)$ exhibits significant differences between $t > 0$ and $t < 0$, both integrals equal zero for all t.

Keywords: Painlevé I hierarchy; KdV equation; full asymptotic expansion; total integrals; Riemann-Hilbert method.
AMS Classification: 33C10, 33E17, 34M55, 41A60.

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Multiple Orthogonal Type I Polynomials on the Askey Scheme

Monday 24
17:00
Room C

Amílcar Branquinho, Juan Díaz, Ana Foulquié Moreno, and Manuel Mañas

Abstract

Multiple orthogonal polynomials are a generalization of usual orthogonal polynomials that arises from considering orthogonality respect to, not one, but an arbitrary number of weight functions $w_1, \cdots, w_p$. This leads to two different kinds of polynomials. On one hand there are the multiple type II polynomials, that have been widely studied and many families are already known. On the other hand there are the multiple type I polynomials. These ones have been less studied and not so many families are known. Here, such lack of knowledge is broken by providing explicit expressions for the type I polynomials corresponding to the discrete families of Hahn, Meixner, Kravchuk and Charlier; all of them considering systems of $p = 2$ weight functions, cf. [1]. Then, a further step is given to find the type I polynomials corresponding to the classical families of Jacobi–Piñeiro, Laguerre and Hermite; this time for an arbitrary number $p \geq 2$ of weight functions, cf. [2] for Jacobi–Piñeiro and Laguerre. All of these expressions were unknown so far despite the corresponding type II polynomials for these families being known from twenty, or even more, years ago. Finally, all of them are expressed through special functions such as the generalized hypergeometric series or the Kampé de Fériet series.

Keywords: Multiple orthogonal polynomials of type I, generalized hypergeometric functions, Kampé de Fériet functions, AT systems, Askey scheme, Hahn, Meixner, Kravchuk, Charlier, Jacobi–Piñeiro, Laguerre, Hermite
AMS Classification: 33C45, 33C47, 42C05, 47A56.

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Classical orthogonal polynomials. One variable vs two variables

Monday 24
15:30
Room A

HERBERT DUEÑAS RUIZ, ALEJANDRA PÉREZ SÁNCHEZ

ABSTRACT

We present some connections between classical orthogonal polynomials in one variable, and a type of bivariate polynomials defined as a product of the univariate ones. We study the form of the partial differential equations (PDE) in terms of the ordinary differential equations (ODE) satisfied by the classical orthogonal polynomials in one variable. Additionally, we present some examples in one and two variables programmed in Matlab, and the differential equations that these polynomials satisfy.

Keywords: Classical Orthogonal Polynomilas, Bivariate Polynomials, Inner Products, Differential Equations, Partial Differential Equations.
AMS Classification: 33C47, 42C05.

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Tuesday 25
11:30
Room A

Computation in terms of elementary functions of an integral from the Elastodynamic Theory

Chelo Ferreira, José Luis López and Ester Pérez Sinusía

The integral $\int_0^\infty \frac{J_\mu(rt)J_\nu(Rt)}{t^\alpha(t-s)}dt$ plays an essential role in the study of several phenomena in the theory of elastodynamics [[1], 2014]. But an exact evaluation of this integral in terms of elementary functions are barely known. In this paper, we derive two analytic representations of this integral in the form of convergent series of elementary functions and hypergeometric functions. These series have an asymptotic character for either, small values of the variable s, or for small values of the variables r and R. They are derived by using the asymptotic technique designed in [[2], 2008] for Mellin convolution integrals. Some numerical experiments show the accuracy of the approximations.

Keywords: Convergent expansions; Asymptotic expansions; Bessel functions.
AMS Classification: 33C05, 41A58, 41A80.

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Bidiagonal Factorization of Hessenberg matrices from multiple orthogonal polynomials

Tuesday 25
12:30
Room C

Amílcar Branquinho, Juan Díaz, Ana Foulquié Moreno, and Manuel Mañas

Abstract

Recently, in [2] a Shohat-Favard theorem has been proved for banded bounded matrices that have a positive bidiagonal factorization. In [1] it is established conditions, expressed in terms of continued fractions, under which an oscillatory tetradiagonal Hessenberg matrix can have such a positive bidiagonal factorization. In [3] it is explored the bidiagonal factorization of the recurrence matrix of Hahn multiple orthogonal polynomials, in the case of 2 weights. The factorization is expressed in terms of ratios involving the generalized hypergeometric function ${}_3F_2$ and is proven using recently discovered contiguous relations.

In this talk, using as a departure point the Hessenberg matrix associated to the recurrence relation in the step line of multiple orthogonal polynomials, we show how to obtain this bidiagonal factorization coming from the coefficients of the type I multiple orthogonal polynomials. As a case study we obtain the bidiagonal factorization related to the Jacobi-Piñeiro multiple orthogonal polynomials for p weights. Finally, using limit relations we can obtain this bidiagonal factorization for multiple Laguerre of kind I.

Keywords: Multiple orthogonal polynomials, hypergeometric series, Hessenberg matrices, recursion matrix, recurrence, Hahn, Laguerre, Meixner, Jacobi-Piñeiro, AT systems
AMS Classification: 42C05, 33C45, 33C47, 47B39, 47B36.

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Harmonic analysis associated with generalized differential Bessel operator

Tuesday 25
12:00
Room A

Dhouha Gaied, Moncef Dziri

Abstract

We investigate harmonic analysis associated with the generalized Bessel differential operator $\Delta_{\alpha_0,\alpha_1}$ defined for $\alpha_1 > -2$ and $\alpha_0 > \max\left(-\frac{\alpha_1}{2}, 0\right)$ by $\Delta_{\alpha_0,\alpha_1} = x^{-\alpha_1}\ell_{\frac{\alpha_0-1}{2}}$, with ℓ_α is the Bessel differential operator. More precisely, we establish the key of this study, which is the product formula of the eigenfunctions of this operator. We define and study the Riemann-Liouville and Weyl transforms associated with $\Delta_{\alpha_0,\alpha_1}$.

Keywords: Bessel functions; differential equation; Fourier transform; convolution product; translation; inversion formula; Integral transforms.
AMS Classification: 33C10;42A38;43A62.

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Nonlinear Analysis and Geometry Laboratory (LR21LS08).

Thursday 27
11:30
Room C

On the Number of components of random polynomial lemniscates

Subhajit Ghosh

Abstract

A lemniscate of a complex polynomial Q_n of degree n is a sublevel set of its modulus, i.e., of the form $\{z \in \mathbb{C} : |Q_n(z)| < t\}$ for some $t > 0$. The study of lemniscates was pioneered by Erdős, Herzog, and Piranian in [1], where they asked various questions regarding the number of connected components of a unit lemniscates (i.e. for $t = 1$). In general, the number of components of a unit lemniscate can vary anywhere between 1 and n. However, for random polynomials with roots chosen from the certain probability measure, numerical simulations show a giant component alongside some tiny components. In this paper, we quantify this numerical observation. First, we show that the expected number of connected components of lemniscates whose defining polynomial has i.i.d. roots chosen uniformly from $\mathbb{D}$, is bounded above and below by a constant multiple of $\sqrt{n}$. On the other hand, if the i.i.d. roots are chosen uniformly from $\mathbb{S}^1$, we show that the expected number of connected components, divided by n, converges to $\frac{1}{2}$. Drawing tools from [2], we also summarize a phase transition phenomena for number of components if the underlying probability measure is $r\mathbb{D}$, $r\mathbb{S}^1$ respectively.

Keywords: Polynomial Lemniscates, Pairing of Zeroes and Critical Points, Logarithmic Potentials, Concentration Inequalities.
AMS Classification: Primary 60D05, Secondary 30C15.

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Classification of exceptional Jacobi polynomials

Thursday 27
11:30
Room A

Mariangeles García-Ferrero, David Gómez-Ullate, Robert Milson

Abstract

The close connection between Darboux transformations and exceptional orthogonal polynomials (XOPs) was established in [1], proving a conjecture previously formulated in [2]. This paper paved the way for a full classification of XOPs, but this task proved to be harder than initially envisaged due to the discovery of whole new families of exceptional polynomials with an arbitrary number of continuous parameters. These new families exploit a certain degeneracy that occurs for certain values of the parameters in Laguerre and Jacobi polynomials. Once this mechanism has been properly studied, a full classification of exceptional Jacobi polynomials that includes both the "traditional" and the new exceptional families can be attained. If the examples given in [3] and [4] can be seen as continuous deformations of classical polynomials (Legendre and ultraspherical, respectively), the most general class of the new exceptional Jacobi families can be seen as a continuous deformation of the previously known exceptional Jacobi families indexed by two partitions, [5]. The whole classification is based on establishing a correspondence between spectral diagrams and operators, and it will be further explained in the talk presented by Robert Milson.

Keywords: Exceptional Jacobi polynomials, Degenerate Darboux transformations, Continuous deformations.
AMS Classification: 33C47; 34L10; 34A05

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Calculation of the relaxation modulus in Andrade model by using the Laplace transform

Thursday 27
16:30
Room C

González-Santander J. L., Spada G. and Mainardi F.

Abstract

In the framework of the theory of Linear Viscoelasticity, we calculate the relaxation modulus in Andrade model for the case of a rational parameter $\alpha = m/n \in (0,1)$ in terms of Mittag-Leffler functions. It turns out that the expression obtained can be rewritten in terms of Rabotnov functions. Moreover, for the original parameter $\alpha = 1/3$ in Andrade model, we obtain an equivalent expression in terms of Miller-Ross functions, and the asymptotic behaviours for $t \to 0$ and $t \to \infty$. All the derived expressions have been numerically checked by using Talbot's method for the inverse Laplace transform.

Keywords: Mittag-Leffler function, Laplace transform, Linear Viscoelasticity.
AMS Classification: 33E12, 44A10.

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Regular multi-parametric rational extensions of the trigonometric Darboux-Pöschl-Teller potential

Thursday 27
15:30
Room A

Yves Grandati and Christiane Quesne

Abstract

By using chains of Darboux transformations based on para-Jacobi polynomials, we build multistep rational extensions of the trigonometric Darboux-Pöschl-Teller potential which depend on families of continuous "times" parameters. We obtain sufficient conditions on the times parameters to ensure regularity of the extended potentials and give some illustrations of the parameter dependence of the extended potentials.

Keywords: Orthogonal Polynomials, Exceptional Orthogonal polynomials, Exactly solvable quantum systems, Supersymmetric Quantum Mechanics
AMS Classification: 33C45, 33C47, 81Q05, 81Q60.

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Monday 24
16:00
Room A

Multivariate q-Racah polynomials as stochastic duality functions

Wolter Groenevelt and Carel Wagenaar

Abstract

In studying interacting particle processes stochastic duality can be very useful: it can be used to study a process through a simpler dual process. In recent years several families of orthogonal polynomials have been shown to appear as stochastic duality functions. In this talk I will consider an interacting particle process, generalized dynamic ASEP, which is a generalization of the standard asymmetric simple exclusion process (ASEP). Using representation theory of $\mathcal{U}_q(\mathfrak{sl}_2)$ it can be shown that the process is (almost) self-dual with multivariate q-Racah polynomials as duality functions. By taking appropriate limits dualities between several other interacting particle processes can be obtained.

Keywords: q-Racah polynomials, stochastic duality, representation theory

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On zero behavior of higher-order discrete Sobolev q-Hermite I orthogonal polynomials

Thursday 27
16:00
Room B

Edmundo J. Huertas, Alberto Lastra, Anier Soria-Lorente, and Víctor Soto-Larrosa

Abstract

In this work, we investigate the sequence of monic q-Hermite I-Sobolev type orthogonal polynomials of higher-order, denoted as $\{\mathbb{H}_n(x;q)\}_{n\geq 0}$, which are orthogonal with respect to the following non-standard inner product involving q-differences

$$\langle p,q\rangle_\lambda=\int_{-1}^{1} f\,(x)\,g\,(x)\,(qx,-qx;q)_\infty d_q(x)+\lambda\,(\mathcal{D}_q^j f)(\alpha;q)\mathcal{D}_q^j g)(\alpha;q),$$

where $\alpha\notin(-1,1)$, λ belongs to the set of positive real numbers, $\mathcal{D}_q^j$ denotes the j-th q-discrete analogue of the derivative operator, and $(qx,-qx;q)_\infty d_q(x)$ denotes the orthogonality weight with its points of increase in a geometric progression. Among other analytic results, for real values of $q^j\alpha\notin(-1,1)$, we present sharp bounds and a comprehensive analysis of the asymptotic behavior of their zeros, as the parameter λ varies from zero to infinity.

Keywords: q-Hermite I Sobolev-type orthogonal polynomials, q-hypergeometric series.
AMS Classification: 33C45, 33C47.

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On the zeros of quasi-orthogonal Meixner polynomials

Wednesday 26
12:00
Room C

Aletta Jooste

Abstract

In his 1934 paper [4], Josef Meixner classifies five classes of orthogonal polynomials, p_n, with generating function $f(t)e^{xu(t)} = \sum_{n=0}^{\infty} c_n p_n(x) t^n$, which include the Meixner polynomials. The Meixner polynomials $M_n(x;\beta,c)$ are orthogonal on the positive real line for $\beta > 0$ and $c \in (0,1)$, with respect to a discrete weight function. We discuss results on the zeros of these polynomials that were published in three different papers [1, 2, 3], which include results on the location of the first few zeros of the quasi-orthogonal order 1 and quasi-orthogonal order 2 Meixner polynomials.

Keywords: Orthogonal Polynomials, Meixner polynomials, quasi-orthogonality, zeros

AMS Classification: 33C45

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Thursday 27
16:00
Room C

Monotonicity and unimodality of some functional series ratios

Dmitrii Karp, Anna Vishnyakova and Yi Zhang

Abstract

Elementary, but very useful lemma due to Biernacki and Krzyż (1955) asserts that the ratio of two power series inherits monotonicity from the monotonicity of ratios of the corresponding power series coefficients. Over the last two decades it has been realized that a similar claim holds for unimodality of power series ratios (under some additional technical conditions). In the talk, we discuss generalization of these properties to ratios of more general functional series, in particular, factorial and inverse factorial series. The key role in this considerations is played by the notion of total positivity. Finally, we illustrate our general results exhibiting certain statements on monotonicity patterns for ratios of some special functions.

Keywords: Monotonicity, unimodality, power series, factorial series
AMS Classification: 26D15, 33C20, 33D15.

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Zeros of orthogonal and paraorthogonal polynomials via CMV matrices

Rostyslav Kozhan

Tuesday 25
16:00
Room A

Abstract

We study asymptotics of zeros of orthogonal $\Phi_{n,N}(z)$ and paraorthogonal $\Phi_{n,N}^{(\beta)}(z)$ polynomials on the unit circle when the Verblunsky coefficients $\alpha_{n,N}$ depend on a parameter N.

We establish the asymptotic zero counting measures under the condition that $\alpha_{n,N}$ has a limit when $n/N \to s$ for almost all s or if $\alpha_{n,N}$ is asymptotically periodic. This can be viewed as an analogue of the results of [1, 2, 5] (see also [3, 4]) for the unit circle.

We provide two different proofs relying on the operator theory of CMV matrices and the ratio asymptotics of orthogonal polynomials.

This is a joint work with František Štampach.

Keywords: Orthogonal Polynomials on the Unit Circle, Paraorthogonal polynomials, CMV Matrices, Zeros, Varying Orthogonality, Ratio Asymptotics

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Tuesday 25
12:00
Room C

Orthogonal polynomials in a normal matrix model with two insertions

Mario Kieburg, Arno Kuijlaars, and Sampad Lahiry

Abstract

The talk is about the asymptotic behavior of polynomials $P_{n,N}$ with orthogonality in the complex plane

$$\int_{\mathbb{C}} P_{n,N}(z)\overline{z}^k |z^2+a^2|^{2cN} e^{-N|z|^2} dA(z) = 0, \quad k = 0, \ldots, n-1,$$

with $c, a > 0$ and $dA(z)$ denotes planar Lebesgue measure. These polynomials are connected with a normal matrix model with external potential $N|z|^2 - 2cN \log|z^2+a^2|$ which is a modification of the Ginibre ensemble with two logarithmic singularities [3]. The eigenvalues of the random matrices fill out a bounded region in the complex plane as $n, N \to \infty$ with $n/N \to t > 0$. We prove, that for a certain regime of parameters a, c, t, the zeros of the orthogonal polynomials tend to an interval on the real line, with an asymptotic density that is characterized by a vector equilibrium problem.

Our analysis essentially relies on the reformulation of the planar orthogonality as non-Hermitian multiple orthogonality [1, 2] and on a steepest descent analysis of the associated Riemann-Hilbert problem [4] of size 3×3.

Keywords: Planar Orthogonal Polynomials, Multiple Orthogonal Polynomials, Riemann-Hilbert Problem, Vector Equilibrium Problem
AMS Classification: 30C10, 31A05, 42C05.

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Monday 24
12:30
Room C

Bidiagonal matrix factorisations associated with symmetric multiple orthogonal polynomials and lattice paths

Hélder Lima

Abstract

This talk is based on the preprint with the same title available at arXiv:2308.03561 [math.CA].

In the two main results of this talk, we present representations as products of bidiagonal matrices for the Hessenberg matrices associated with the components of the decomposition of m-fold symmetric multiple orthogonal polynomials and we show that these Hessenberg matrices are production matrices of the generating polynomials of certain lattice paths. Therefore, we can use the recently found connection of multiple orthogonal polynomials with lattice paths and branched continued fractions to study m-fold symmetric multiple orthogonal polynomials.

We revisit known results about the location and interlacing of the zeros and the existence of orthogonality measures supported on a star-like set on the complex plane for m-fold symmetric multiple orthogonal polynomials with positive recurrence coefficients. Then, we show that the components of the decomposition of these polynomials are multiple orthogonal with respect to measures on the positive real line and we find combinatorial interpretations for the moments of the orthogonality measures. Finally, we give explicit formulas as terminating hypergeometric series for Appell sequences of m-fold symmetric multiple orthogonal polynomials with respect to an arbitrary number of measures on a star-like set whose densities can be expressed via Meijer G-functions on the positive real line.

Keywords: Multiple orthogonal polynomials, m-fold symmetry, Hessenberg matrices, bidiagonal matrices, production matrices, lattice paths, hypergeometric series, Appell polynomials.
AMS Classification: 15A23, 15B48, 33C45, 42C05 (primary);
05A19, 30C15, 30E05, 33C20 (secondary).

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The saddle point method for double integrals

José L. López

Wednesday 26
11:30
Room B

Abstract

Classical asymptotic methods for integrals were first designed for one-dimensional integrals and later generalized to double and multiple integrals. One important exception is the well-known saddle point method for integrals on complex contours, method that is very useful in the approximation of special functions. A standard saddle point method for double integrals does not exist. This is so because it is not possible to extend the concept of "steepest descent path" used in the one-dimension to something like a "steepest descent surface" that should be used on double integrals. And this is not possible simply because such a concept does not exist [1]. But this is not the end of the story: there is a simplified version of the saddle point method for one-dimensional integrals, the "simplified saddle point method" [2], that can be generalized to double integrals. In the simplified method, the "steepest descent path" is no longer needed, as this curve is replaced by something simpler: a "descent straight". And it turns out that this simpler concept can be generalized to double integrals, becoming a "descent plane". As in the one-dimensional case, the descent plane for double integrals can be systematically computed for any integral, that is, there is always a descent plane. Then, with this idea, we can give the first steps to design of a "simplified saddle point method" for double integrals.

Keywords: double integrals, saddle point method, asymptotic expansions.
AMS Classification: 33E20, 41A58.

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Thursday 27
16:30
Room B

Explicit representations of the norms of the Jacobi-Sobolev, Laguerre-Sobolev and Gegenbauer-Sobolev polynomials

Clemens Markett

Abstract

One of the intrinsic features of the discrete Jacobi-, Laguerre- and Gegenbauer-Sobolev orthogonal polynomials is their L^2-norm in the corresponding inner product spaces. The corresponding inner products are built upon a classical measure on an interval of the real line and two point masses $N, S > 0$ at a finite endpoint of the interval involving functions and their first derivatives. Among other applications, the knowledge of these norms is essential for orthonormalizing the polynomials and thus indispensable for any orthogonal expansion.

In this talk we present explicit representations of the squared norms for the three stated Sobolev classes as well as for the so-called Jacobi-type polynomials, where the inner product is equipped with two in general different point masses $M, N > 0$ at $x = \pm 1$. In all these cases, the proof proceeds from an appropriate representation of the polynomials each consisting of components reflecting their dependence on the point masses. Surprisingly it turns out that all the resulting pieces can be combined to a very elegant form based on two essentially identical factors varying only in a small shift of the polynomial degree n. Moreover, each factor consists of four terms similar to the polynomials themselves. In particular, the asymptotic behavior for large n can be read off.

Finally, we establish various interrelations between the presented norms.

Keywords: Sobolev Orthogonal Polynomials, Inner Product Space, Orthonormalization
AMS Classification: 33C47, 42C10.

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Banded total positive matrices and multiple orthogonality: Favard, Gauss quadrature and normality

Monay 24
15:30
Room C

Manuel Mañas, Amílcar Branquinho, and Ana Foulquié

Abstract

We present a spectral Favard theorem for mixed multiple orthogonality in terms of positive bidiagonal factorizations of bounded band matrices. We also present a Gauss quadrature formula and corresponding precision indices. Finally we prove normality at all degrees in the step-line for banded totally positive recurrence matrices.

Keywords: Orthogonal Polynomials, Approximation Theory, Applications
AMS Classification: 33CXX, 42CXX.

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Agencia Estatal de Investigación PID2021-122154NB-I00

Tuesday 25
12:30
Room B

Recurrence relations for Catalan polynomials and generating functions for bounded operators

Pedro J. Miana and Natalia Romero

Abstract

Let $c = (C_n)_{n\geq 0}$ be the Catalan sequence and T a linear and bounded operator on a Banach space X such $4T$ is a power-bounded operator. The Catalan generating function is defined by the following Taylor series,

$$C(T) := \sum_{n=0}^{\infty} C_n T^n.$$

Note that the operator $C(T)$ is a solution of the quadratic equation $TY^2 - Y + I = 0$. In this talk we define powers of the Catalan generating function $C(T)$ in terms of the Catalan triangle numbers. We define the Catalan polynomials using Catalan triangle numbers and we show some recurrence formulae. The spectrum of c^{*j} and the expression of c^{-*j} for $j \geq 1$ in terms of Catalan polynomials is obtained (here $*$ is the usual convolution product in sequences). Finally, we give some particular examples to illustrate our results and some ideas to continue this research in the future. This is a joint paper with Natalia Romero (Universidad de La Rioja).

Keywords: Recurrence relations, Catalan numbers, bounded operators.
AMS Classification: 11B37, 33CXX, 47B38.

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Spectral diagrams and exceptional polynomials

Thursday 27
16:30
Room A

Mariangeles García-Ferrero, David Gómez-Ullate,
Robert Milson

Abstract

Exceptional orthogonal polynomials are generalizations of classical OP in as much as they arise as solutions of second-order Sturm-Liouville problems. However, unlike their classical counter-parts the families in questions consist of polynomials that are missing a finite number of degrees.

Most of the literature on the theory of exceptional polynomials and operators on the past 10 years has focused on the polynomial spectrum and eigenfunctions. It is now clear that that a complete, systematic approach needs to consider the larger class of quasi-rational (log-derivative is rational) eigenfunctions. Indeed, we argue that a proper characterization of the quasi-rational eigenfunctions of a given exceptional operator and their asymptotic behaviour essentially specifies the operator uniquely. For this reason, we introduce the the key concepts of a *quasi-rational spectrum* and a *spectral diagram*.

I will report on some recent work that allows for the full classification of exceptional Jacobi polynomials, including a novel subclass that allows for an arbitrary number of continuous parameters. The classification methodology reduces to two principles: (i) a combinatorial description of all possible spectral diagrams; and (ii) a procedure for mapping a given spectral diagram into the corresponding exceptional operator.

This initial classification is too coarse because the orthogonality of most of the resulting families is formal in as much as the integrals in questions are divergent and the corresponding inner products are semi-definite. Time permitting, I will also discuss the methodology for tackling this obstacle. This consists of (a) defining a procedure for regularizing the divergent integrals in questions, and then (b) a Theorem to the effect that the subclass of regular families can be characterized and classified in terms of a norm-positivity condition.

Keywords: Exceptional Jacobi polynomials, Degenerate Darboux transformations, Continuous deformations.
AMS Classification: 33C47; 34L10; 34A05

This research has been financed in part by MCIN/AEI/10.13039/501100011033 under grants PID2021-122154NB-I00 and TED2021-129455B-I00, abd by the EU under the program NextGenerationEU/PRTR

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Sheffer-Dunkl sequences via umbral calculus in the Dunkl context

Alejandro Gil Asensi, Judit Mínguez Ceniceros and Juan Luis Varona

Tuesday 25
12:00
Room B

Abstract

Umbral calculus refers to a series of techniques that can be used to prove some polynomial formulas. Nowadays, it mostly involves the study of Sheffer sequences. In this paper, we focus on a generalization of umbral calculus in a Dunkl context (that we call Dunkl-umbral calculus). Here, the operators of classical umbral calculus are naturally replaced by operators of the Dunkl theory on the real line. In this context we define for the first time Sheffer-Dunkl sequences, $\{s_{n,\alpha}(x)\}_{n=0}^{\infty}$, and provide some properties and examples. We also connect, via Dunkl-umbral calculus, properties of some polynomials in a Dunkl sense that have appeared in the literature in the recent years, like Bernoulli-Dunkl or Euler-Dunkl polynomials ([1, 2, 3, 4]).

Keywords: Approximation Theory, Applications, Sheffer-Dunkl sequences, Dunkl-umbral calculus.
AMS Classification: 11B83, 05A40, 11B68.

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Multiple Orthogonal Poynomials and Finite Free Convolution

Monday 24
16:00
Room C

Rafael Morales, Andrei Martinez-Finkelshtein and Daniel Perales

Abstract

Some multiple orthogonal polynomials can be wrote explicitly as terminating generalized hypergeometric functions. However, extracting the information about their zeros from this fact is not trivial. In this talk, we address some recently discovered applications of the notion of the free finite convolution of polynomials (developed in the framework of the free probability theory) to the study of the asymptotic and properties of zeros of hypergeometric polynomials. In particular, we discuss some consequences for multiple orthogonal polynomials.

Keywords: Multiple Orthogonal Polynomials,Finite Free Convolution, Zeros, Asymptotic
AMS Classification: 33C20, 33C45.

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Thursday 27
17:00
Room C

Coulomb Gases in an Ellipse and Orthogonal Polynomials

Taro Nagao

Abstract

Using orthogonal polynomials on the complex plane, one is able to analyze two-dimensional Coulomb gases at a special temperature. This method is borrowed from the theory of random non-hermitian matrices, and gives the determinant forms of the gas molecule correlation functions. If the Coulomb gas is in a circle, the corresponding orthogonal polynomials are monomials, and if it is in an ellipse, we need to employ the Chebyshev polynomials[1]. More recently Coulomb gas systems in an ellipse with some special one-body potentials are found to be associated with some types of the Jacobi polynomials[2, 3]. In this presentation, some gas molecule correlation functions are evaluated in the thermodynamic limit by utilizing such relationships between Coulomb gases in an ellipse and the classical orthogonal polynomials.

Keywords: Orthogonal Polynomials, Coulomb Gases, Random Matrices
AMS Classification: 33C45, 82D05, 60B20.

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Plancherel and inversion formulas for the Dunkl-type Segal-Bargmann transform

Tuesday 25
11:30
Room B

Fethi Soltani and Meriem Nenni

Abstract

In 1961, Bargmann introduced the classical Segal-Bargmann transform and in 1984, Cholewinsky introduced the generalized Segal-Bargmann transform. These two transforms are the aim of many works, and have many applications in mathematics. In this paper we introduce the Dunkl-type Segal-Bargmann transform $\mathscr{B}_\alpha$ associated with the Coxeter group $\mathbb{Z}_2^d$. Next, we investigate for this transform the main theorems of harmonic analysis (Plancherel and inversion formulas). Finally, we study some local uncertainty principles associated with the transform $\mathscr{B}_\alpha$.

Keywords: Dunkl-type Segal-Bargmann transform, Plancherel and inversion formulas, local-type uncertainty inequalities, dispersion inequality.
AMS Classification: 32A15; 44A05; 44A20

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LR Analyse Mathematics and Applications (LR11ES11)

A mixed interpolation-regression method using zeros of orthogonal polynomials and its applications

Tuesday 25
16:30
Room B

Francesco Dell'Accio, Francisco Marcellán, Federico Nudo

Abstract

The Constrained Mock-Chebyshev Least Squares approximation (CMCLS approximation) is a novel method designed to address the Runge phenomenon in interpolating a function f on a grid X_n formed by $n+1$ equidistant points. It exclusively interpolates the function f at the nodes of X_n closest to the Chebyshev–Lobatto nodes of degree $m = \mathcal{O}(\sqrt{n})$, while leveraging the remaining nodes to enhance the accuracy through a simultaneous regression.

This work extends the CMCLS approximation by proposing a comprehensive strategy for defining an interpolation-regression operator on equally spaced nodes. This operator is based on interpolating the function f at the points of X_n which emulate the behavior of zeros of general orthogonal polynomials of degree m, thereby harnessing their advantageous properties. The remaining nodes of X_n are strategically employed to further improve the accuracy of the approximation through a simultaneous regression. Given the widespread applications of the CMCLS operator, this work also aims to introduce new stable quadrature and numerical differentiation formulas using the new interpolation-regression operator, both of which are easily adaptable to the multivariate scenario.

Keywords: Orthogonal Polynomials, Approximation Theory, Applications
AMS Classification: 05E35, 47A58.

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Wednesday 26
12:00
Room B

A generalization of the Laplace's method for integrals

Pablo Palacios, José L. López, Pedro J. Pagola

Abstract

In [1] a modification of the Laplace's method for deriving asymptotic expansions of Laplace integrals was introduced. This modification simplifies the computations, giving explicit formulas for the coefficients of the expansion. On the other hand, motivated by the approximation of special functions with two asymptotic parameters, Nemes [2] has generalized Laplace's method by considering Laplace integrals with a linear dependence of the phase function on two asymptotic parameters of a different asymptotic order.

In this talk, we investigate if the simplifying ideas introduced in [1] for Laplace integrals with one large parameter may also be applied to the more general Laplace integrals considered in Nemes's theory. We show not only that the answer is yes, but also that those simplifying ideas can be applied to more general Laplace integrals where the phase function depends on the asymptotic variable in a more general way. We derive new asymptotic expansions for this more general kind of integrals with simple formulas for the coefficients of the expansion. In particular, this theory can be applied to special functions with two or more large parameters of a different asymptotic order. We give some examples of special functions that illustrate the theory.

Keywords: Asymptotic Expansions, Special Functions
AMS Classification: 41A60, 41A58

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Tuesday 25
12:30
Room A

On the number of complex zeros of derivatives of Bessel functions of the first kind

Seok-Young Chung, Sujin Lee and <u>Young Woong Park</u>

Abstract

Is is well-known that the zeros of J_ν are real when $\nu > -1$. Using Lommel polynomials, Hurwitz showed that J_ν has $2s$ pairs of complex zeros when $-s-1 < \nu < -s$, $s \in \mathbb{N}$. With respect to real zeros, similar results were obtained for the derivatives of Bessel functions. For example, J'_ν has only real zeros when $\nu > 0$. However, not much is known for their complex zeros. Considering a variation of Lommel polynomials and its Hankel determinant, we investigate the number of complex zeros of J'_ν when ν is negative. In particular, the bifurcation points of the number of complex zeros turn out to be the zeros of $J'_\nu(\nu)$ as well as negative integers.

Keywords: Bessel Functions, Lommel Polynomials, Complex Zeros
AMS Classification: 33C10, 33C45.

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Wednesday 26
12:00
Room A

Generalized Bernstein functions and applications to special functions

Henrik Laurberg Pedersen and Stamatis Koumandos

Abstract

Motivated by examples such as incomplete gamma and beta functions we introduce the class $\mathcal{B}_\lambda$ of generalized Bernstein functions of positive order λ.

A function f defined on the positive half-line belongs to $\mathcal{B}_\lambda$ if $x^{1-\lambda}f'(x)$ is a completely monotonic function, i.e. is a smooth function g defined on the positive half-line for which $(-1)^n g^{(n)}$ is positive for all $n \geq 0$.

We investigate fundamental properties of the classes $\mathcal{B}_\lambda$ including their relation to generalized Stieltjes functions of order λ and approximating subclasses.

Examples include incomplete gamma functions, Lerch's trancendent and hypergeometric functions.

This is based on joint work with Stamatis Koumandos, see [1] and [2].

Keywords: Laplace transform, completely monotonic function, Generalized Stieltjes function
AMS Classification: 44A10, 26A48, 33B15, 33C05.

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New analytic expansions of the Lerch trascendent

Ester Pérez Sinusía, Chelo Ferreira and José L. López

Wednesday 26
12:30
Room B

Abstract

We obtain new analytic expansions of the Lerch trascendent function $\Phi(z, s, a)$ [19, Eq. 25.14.1]. The starting point is the integral representation

$$\Phi(z, s, a) = \frac{1}{\Gamma(s)} \int_0^\infty \frac{x^{s-1} e^{-ax}}{1 - ze^{-x}} \, \mathrm{d}x,$$

with $\Re s > 0, \Re a > 0$ if $z \in \mathbb{C} \setminus [1, \infty)$. Two different analytical methods for the approximation of integral transforms of the form $F(z) = \int_0^1 h(t, z) g(t) dt$ are applied. The first method considers a multi-point Taylor expansion of the function $g(t)$ in certain selected points in $[0, 1]$, and provides a uniform convergent expansion of $F(z)$ in a region D of the complex plane where the function $h(t, z)$ can be bounded independently of z. The second method considers the multi-point Taylor expansion of $h(t, z)$ at certain appropriately selected base points, obtaining larger domains of convergence for the series expansion of $F(z)$ and with the property of being uniformly valid in compact sets of the $z-$complex plane. Numerical experiments illustrate the accuracy of the new approximations.

Keywords: Lerch trascendent function, Approximation Theory, Uniform Convergent Expansions, Special Functions
AMS Classification: 33E20, 41A58, 41A80.

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Zernike Sobolev type Polynomials. Quadrature and Interpolation.

Monday 24
16:30
Room A

Gabriel Arturo Pulido, Herbert Dueñas Ruiz

Abstract

We present the family of classical orthogonal polynomials over the unit ball, focusing on the family of Zernike orthogonal polynomials. We discuss some applications in optical systems and various Zernike Sobolev-type inner products. We offer numerous concrete examples illustrating various Zernike Sobolev type polynomials, juxtaposed with their unperturbed counterparts. We demonstrate a special type of quadrature and interpolation found in recent literature for functions defined on the unit disk, using Zernike polynomials, where one of the main results is the following.

$$\int_D Z^l_{N,n}(x)\,dx = \sum_{i=1}^{m} R_{N,n}(r_i)\omega_i \sum_{j=1}^{2m} \frac{2\pi}{2m} S^l_N(\theta_j),$$

where $Z^l_{N,n}(x)$ is the family of Zernike polynomials of degree $N+2n$, $R_{N,n}(r)$ is a radial polynomial, and $S^l_N(\theta)$ is an angular function.

Finally, we present an analogous representation with the Zernike Sobolev family and the representation of Zernike in higher dimensions.

Keywords: Orthogonal Polynomials, Polynomials in Several Variables, Sobolev inner Products, Zernike Polynomials, Gaussian Quadrature, Interpolation.
AMS Classification: 33C50, 42C05.

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The Riemann-Hilbert problem for Krall orthogonal polynomials

Thursday 27
17:00
Room A

Alejandro Quintero-Roba and Andrei Martinez-Finkelshtein

Abstract

In this talk, we give background on the Riemann-Hilbert problem (RHp) for orthogonal polynomials (OP) and its versatility in finding general properties of specific OP families such as the ordinary differential equation the family satisfies. We also discuss the Krall-Legendre family of orthogonal polynomials, which is known for satisfying a fourth-order differential equation. Then, we formulate the RHp for the Krall-Legendre OP, prove the existence and uniqueness of its solution, and show the method to obtain the first-order matrix ODE, and the second-order scalar ODE for the Krall-Legendre OP, as a first approach to finding the fourth-order scalar ODE from the Riemann-Hilbert formulation, as our final goal.

Keywords: Orthogonal Polynomials, Riemann-Hilbert problem, Fourth-Order Differential Equation.
AMS Classification: 33C47, 42C05.

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Thursday 27
17:00
Room B

Deformed Laguerre-Hahn orthogonal polynomials on the real line

Maria das Neves Rebocho

Abstract

In this talk we focus on the Laguerre-Hahn class on the real line, that is, the sequences of orthogonal polynomials whose Stieltjes functions satisfy a Riccati type differential equation with polynomial coefficients. We shall take Stieltjes functions subject to a deformation parameter, t, and we derive systems of differential equations and give Lax pairs, yielding non-linear differential equations in t for the recurrence relation coefficients and Lax matrices of the orthogonal polynomials. A specialisation to a non semi-classical case obtained via a Möbius transformation of a Stieltjes function related to a modified Jacobi weight is studied in detail, showing that such a system is governed by a differential equation of the Painlevé type P_{VI}.

This talk is based on the paper [1].

Keywords: Orthogonal polynomials; matrix Sylvester equations; modified weight; semi-classical weight; Painlevé equations
AMS Classification: 33C47, 39A99

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The work of MNR is partially supported by CMA-UBI (grants UID-B-MAT/00212/2020 and UID-P-MAT/00212/2020), funded by FCT-Portugal.

Approximation on the unit ball via gradients or Laplacians

Monday 24
17:00
Room A

Marlon J. Recarte, Misael E. Marriaga and Teresa E. Pérez

Abstract

In this work, we study the orthogonal structure on the unit ball $\mathbf{B}^d$ of $\mathbb{R}^d$ with respect to Sobolev inner products by means of multivariate differential operators such as gradients or Laplacians, in two different ways. Our main contribution consists in the study of the approximation properties of the Fourier sums with respect to orthogonal polynomials associated with the Sobolev inner products. Some numerical experiments are given.

Keywords: Orthogonal Polynomials, Approximation on the ball, inner product via Laplacian, inner product via gradient,
AMS Classification: 33C50, 42C05.

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Wednesday 26
11:30
Room C

On the zeros of quasi-orthogonal polynomials

C. Brezinski, K.A. Driver and M. Redivo-Zaglia

polynomials

Abstract

The family of orthogonal polynomials $\{P_n\}$ is defined by the orthogonality conditions

$$\int_a^b x^k P_n(x) w(x)\, dx = 0 \quad \text{for} \quad k = 0, \ldots, n-1,$$

where w is a positive weight function on the finite or infinite interval $[a, b]$. P_n is the polynomial of degree n belonging to the family of orthogonal polynomials on $[a, b]$ with respect to the weight function w.

It is well known that the zeros of P_n are all real and distinct, lie in the interior (a, b) of $[a, b]$, and interlace with those of P_{n+1} and P_{n-1}.

When the orthogonality conditions are satisfied only up to $n - r - 1$ with $r \geq 1$, the polynomials are called *quasi–orthogonal of order* r and some of their zeros escape from the interval $[a, b]$.

In this talk, new results on the location of the zeros of quasi–orthogonal polynomials are given in the cases $r = 1$ and $r = 2$.

Then, these results are applied to Gegenbauer, Jacobi and Laguerre polynomials which are orthogonal with respect to weight functions depending on parameters. When the restrictions on these parameters are not satisfied, we prove that the polynomials are quasi–orthogonal. The corresponding weight functions are investigated and the location of their zeros is discussed.

Keywords: Orthogonal polynomials, Quasi-orthogonal polynomials, Zeros.

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Nonmonic Christoffel perturbations for mixed multiple orthogonal polynomials

Thursday 27
11:30
Room B

Manuel Mañas Baena and Miguel Rojas Rodríguez

Abstract

Consideration is given to mixed multiple orthogonal polynomials and their Christoffel transformations within a general class of non-monic matrix polynomial perturbations, wherein the spectral theorem of matrix polynomials is applicable. Christoffel formulas are derived in instances where tau functions remain non-zero, demonstrating the existence of perturbed mixed multiple orthogonality whenever these tau functions retain their non-zero values.

Keywords: Mixed multiple orthogonal polynomials, Matrix polynomials, Christoffel transformations
AMS Classification: 42C05, 33C47.

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Tuesday 25
16:00
Room B

Chebyshev polynomials with a prescribed zero

Olof Rubin

Abstract

Let E denote a compact subset of the complex plane $\mathbb{C}$ containing an infinite number of points. Then, there exists a unique monic polynomial of degree n that minimizes the infinity norm on E. This polynomial is known as the Chebyshev polynomial associated with E. In this presentation, we will explore a related problem by considering monic minimizers with respect to the infinity norm on the unit circle having a prescribed zero on the boundary. Building upon work in [2], we will see that prescribing a zero on the boundary dramatically changes the behavior of the corresponding minimizers in terms of the asymptotic zero distribution as well as the corresponding norms. Alternatively, this can be viewed as a weighted Chebyshev problem and we will extend the analysis to allow for fractional powers of the boundary zero in order to draw conclusions regarding Chebyshev polynomials corresponding to the lemniscatic family $\{z : |z^m - 1| = 1\}$. Finally, we will supplement our theoretical discussion with numerical experiments conducted using the complex Remez algorithm [4]. These experiments will serve to suggest directions for further study.

Keywords: Chebyshev Polynomials, Approximation Theory, Widom Factors, Zero Distributions
AMS Classification: 41A50. 30C10.

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Weinstein-type Segal-Bargmann transform and its applications to partial differential equations

Thursday 27
12:30
Room C

Fethi Soltani, Hanen Saadi

Abstract

In this work, we give some applications of the Weinstein-type Segal-Bargmann transform $\mathscr{B}_\alpha$ in the field of partial differential equations, such as the time-dependent Schrödinger, diffusion, and heat Cauchy problems associated with the Weinstein operator. The resolution of these types of problems is based on the techniques of the transmutation operators on the Weinstein-type Fock space $F_{\alpha,*}(\mathbb{C}^d)$.

Keywords: Weinstein-type Segal-Bargmann transform; time-dependent Weinstein-Schrödinger equation; time-dependent heat Cauchy problems.
2020 Mathematics Subject Classification: 30H20; 35Q41; 44A20.

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Tuesday 25
15:30
Room C

The Variety Underlying the Askey-Wilson Scheme of Hypergeometric Orthogonal Polynomials

Konrad Schöbel

Abstract

It is well known that the set of all hypergeometric orthogonal polynomials can be organised in the so-called *Askey-Wilson scheme* – a directed acyclic graph with families of hypergeometric orthogonal polynomials as vertices and limits between them as edges. In the topological ordering, we find the generic families of Wilson and Racah polynomials on top and Hermite polynomials as the most degenerate family at the bottom.

In my talk I will show that there is an algebraic variety underlying the Askey-Wilson scheme and how to derive its defining algebraic equations explicitly. Consequently, instead of describing hypergeometric orthogonal polynomials as *many* families, each with *several* parameters, we propose to consider them as a *single* family with *one* parameter that lives on an algebraic variety. This paves the way for an introduction of new and powerful algebraic geometric methods into the theory of hypergeometric orthogonal polynomials, as will be outlined at the end of the talk.

This is an ongoing work in collaboration with Andreas Vollmer (Universität Hamburg, Germany) and Jonathan Kress (University of New South Wales, Australia).

Keywords: hypergeometric orthogonal polynomials, Askey-Wilson scheme, superintegrability, algebraic varieties.
AMS Classification: Primary 33C45; Secondary 14H70, 35N10, 70H33.

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The generalized Littlewood theorem concerning integrals of the logarithm of analytical functions, and its use for analysis of zeroes of analytical functions

Tuesday 25
17:00
Room B

Sergey Sekatskii

Abstract

Recently, we have established and used the generalized Littlewood theorem concerning contour integrals of the logarithm of analytical functions, see e.g. [1 - 7]. Briefly, this Theorem is the following statement connecting the value of the contour integral $\int_C F(z)g(z)\,\mathrm{d}z$ $(z = x+iy)$ of meromorphic functions $g(z)$ and $F(z) = ln(f(z))$ with the location of zeroes $X_\rho^0 + iY_\rho^0$ and poles $X_\rho^{pole} + iY_\rho^{pole}$ of $f(z)$, and residues of $g(z)$ lying inside the contour C, which is a rectangle bounded by the lines $x = X_1$, $x = X_2$, $y = Y_1$, $y = Y_2$:

$$\int_C F(z)g(z)\,\mathrm{d}z$$
$$= 2\pi i \left(\sum_{\rho_g} \mathrm{res}(g(\rho_g) \cdot F(\rho_g)) - \sum_{\rho_f^0} \int_{X_1+iY_\rho^0}^{X_\rho^0+iY_\rho^0} g(z)\,\mathrm{d}z + \sum_{\rho_f^{pol}} \int_{X_1+iY_\rho^{pol}}^{X_\rho^{pol}+iY_\rho^{pol}} g(z)\,\mathrm{d}z \right).$$

This theorem has close connection with the similarly named Littlewood theorem [8, 9], and the proofs of these theorems are also similar; for all necessary technical details see [1, 2].

If asymptotic of the product $g(z)F(z)$ is such that the contour integral value tends to zero in the limit of infinitely large contours C, we obtain $\sum_{\rho_f^0} \int_{-\infty+iY_\rho^0}^{X_\rho^0+iY_\rho^0} g(z)\,\mathrm{d}z - \sum_{\rho_f^{pol}} \int_{-\infty+iY_\rho^{pol}}^{X_\rho^{pol}+iY_\rho^{pol}} g(z)\,\mathrm{d}z = \sum_{\rho_g} \mathrm{res}(g(\rho_g)F(\rho_g))$, and this formula has numerous applications to find different infinite sums and to study zeroes of analytical functions.

Earlier, we apply this Theorem to prove the generalized Li's criterion equivalent to the Riemann hypothesis [3] (see [10, 11]) for original Li's criterion), and attempted to use the obtained results to test the Riemann hypothesis [5]. Recently, this work was highlighted in the entry of the Encyclopedia of Mathematics and its Applications [12].

Later on, we exploited this Theorem to study zeroes of polygamma-, incomplete gamma- and incomplete Riemann zeta-functions [6], as well as elliptical functions [7] and the Hurwitz zeta-function. These results, as well as some new findings, will be presented at the Conference.

For illustration, below we give a few examples.

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1. Generalized Li's criterion equivalent to the Riemann hypothesis is the following Theorem [3]:

Theorem 1 (Generalized Li's criterion)**.** *Riemann hypothesis is equivalent to the non-negativity of all derivatives* $\frac{1}{(n-1)!}\frac{d^n}{dz^n}((z-a)^{n-1}\ln(\xi(z)))\mid_{z=1-a}$ *for all non-negative integers* n *and any real* $a < 1/2$*; correspondingly, it is equivalent also to the non-positivity of all derivatives for all non-negative integers* n *and any real* $a > 1/2$*.*

Here $\xi(z)$ is the Riemann xi-function. The analogue of this Theorem for $a = 1/2$ is considered in [4].

2. The following property of zeroes of the digamma function holds [6].

Theorem 2. *Let* ρ_i *with* $i = 1, 2, 3...$ *be real negative zeroes of digamma function* $\psi(z)$ *arranged in decreasing order, and* $\rho_0 = 1.461632...$ *is the only one positive zero of this function. Then* $\lim_{N\to\infty}(\ln N + \sum_{n=0}^{N}\frac{1}{\rho_i}) = 0$*.*

For similar statements about zeroes of the polygamma functions, see [6].

3. Among numerous formulae describing the sums over inverse powers of zeroes of elliptic functions [7], we present the following. Let ρ_i be zeroes of the Weierstrass function $\wp(z,\tau)$. Then $\displaystyle\sum_{\rho_i}\frac{1}{(\rho_i^{zero})^4} = -10\delta_4$, $\displaystyle\sum_{\rho_i}\frac{1}{(\rho_i^{zero})^6} = -28\delta_6$, $\displaystyle\sum_{\rho_i}\frac{1}{(\rho_i^{zero})^8} = -54\delta_8 + 36\delta_4^2$, etc., where for $j > 0$,

$$\delta_{2j}(\tau) = \sum_{n=-\infty}^{\infty}\sum_{\substack{m=-\infty \\ |m|+|n|\neq 0}}^{\infty}\frac{1}{(m+n\tau)^{2j}}.$$

4. The following Theorem holds.

Theorem 3. *For an arbitrary large positive integer* N *and arbitrary small real* ϵ*, we can find such real value of* $z_0(N,\epsilon)$ *that the function* $\zeta(s,z)$ *with* $|z| \le z_0$ *has at least* N *zeroes in the area* $|s| < \epsilon$*.*

Here $\zeta(s,z)$ is the Hurwitz zeta-function.

5. Finally, and more for the curiosity, we present the following result [6].

Let ρ_i be the roots of equation $f(z) = e^{bz} - a = 0$ having order k_i. Then for $a \neq 1$, $\displaystyle\sum_{\rho_i}\frac{k_i}{\rho_i^2} = \frac{b^2}{(1-a)^2} - \frac{b^2}{1-a}$. If $a = 1$ and $b \neq 0$, we have $\displaystyle\sum_{\rho_i\neq 0}\frac{1}{\rho_i^2} = -\frac{b^2}{12}$. This is simply the statement $\displaystyle\sum_{\substack{n=-\infty \\ n\neq 0}}^{n=\infty}\frac{b^2}{(2\pi ni)^2} = -\frac{b^2}{12}$ (Basel problem solution; quite similarly, we can prove $\zeta(4) = \frac{\pi^4}{90}$, etc). Let now our equation be $f(z) := \exp(bz) - 1 - bz = 0$ with $b \neq 0$. Then $\sum_{\rho_i} \frac{1}{\rho_i^2} = -\frac{b^2}{18}$ (can this be named "the general Basel problem"?), etc.

Keywords: Generalized Littlewood theorem, logarithm of an analytical function, zeroes and poles of analytical function.
AMS Classification: 30E20, 30C15, 33B20, 33B99.

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Polynomial Approximation with Missing Degrees

Thursday 27
12:30
Room A

Brian Simanek

Abstract

The recently discovered families of Exceptional Orthogonal Polynomials highlight the difference between algebraic completeness and analytic completeness among sequences of polynomials. Our main result is a general theorem about polynomial approximation by sequences of polynomials with missing degrees. Some applications will also be discussed. This is based on joint work with Richard Wellman.

Keywords: Polynomial Approximation
AMS Classification: 41A10

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Thursday 27
12:00
Room C

Truncated Freud polynomials

Edmundo J. Huertas, Alberto Lastra, Francisco Marcellán and Víctor Soto-Larrosa

Abstract

We define the family of truncated Freud polynomials $P_n(x;z)$, orthogonal with respect to the linear functional u defined by

$$\langle u, p\rangle = \int_{-z}^{z} p(x)e^{-x^4}dx, \quad p \in \mathbb{R}[x], \quad z > 0.$$

The semiclassical character of $P_n(x;z)$ as polynomials of class 4 is stated. As a consequence, several properties of $P_n(x;z)$ concerning the coefficients $\gamma_n(z)$ in the three-term recurrence relation they satisfy as well as the moments and the Stieltjes function of u are studied. Ladder operators associated with such a linear functional and the holonomic equation that the polynomials $P_n(x;z)$ satisfy are derived. Moreover, an electrostatic interpretation of the zeros of such polynomials as well as the dynamics of the zeros in terms of the parameter z are given.

Keywords: Truncated Freud polynomials, Laguerre-Freud equations, Ladder operators, Holonomic equation, Zeros.
AMS Classification: 42C05; 33C50.

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Wednesday 26
12:30
Room C

Anti-Gaussian quadrature rules related to orthogonality on the semicircle

Marija P. Stanić, Aleksandra Milosavljević and Tatjana V. Tomović Mladenović

Abstract

Let Γ be a unit semicircle $\Gamma = \{z = \mathrm{e}^{\mathrm{i}\theta} : 0 \leq \theta \leq \pi\}$. Orthogonal polynomials on the unit semicircle with respect to the complex-valued inner product

$$\langle f, g\rangle = \int_\Gamma f(z)g(z)(\mathrm{i}z)^{-1}\mathrm{d}z = \int_0^\pi f(\mathrm{e}^{\mathrm{i}\theta})g(\mathrm{e}^{\mathrm{i}\theta})\mathrm{d}\theta$$

was introduced by Gautschi and Milovanović in [1], were the certain basic properties were proved. Such orthogonality as well as the applications involving Gauss-Christoffel quadrature rules were further studied in [2] and [4]. Inspired by Laurie's article [3] for the case of ordinary orthogonality on real line, in this article we introduce anti-Gaussian quadrature rules related to the orthogonality on the semicircle and present a stable numerical method for their construction. Also, some numerical examples are included.

Keywords: Complex-valued inner product, Orthogonal polynomials, Anti–Gaussian quadrature rule
AMS Classification: 65D32, 30C10.

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Thursday 27
15:30
Room C

Recovering orthogonality from the quasi-nature of spectral transformations

A. Swaminathan, Vikash Kumar and Francisco Marcellán

Abstract

We introduce the quasi-orthogonality of polynomials derived from the standard spectral transformations, namely, the Christoffel, Geronimus and Uvarov transformations. We delve into the nonlinear three-term recurrence relation satisfied by these quasi-spectral polynomials of order one. Through the utilization of polynomials generated via canonical spectral transformations, we achieve the recovery of the original orthogonal polynomials by forming linear combinations with quasi-spectral polynomials of order one. By reducing the coefficient degrees in the nonlinear recurrence relation, the orthogonality of quasi-spectral polynomials are established. Recurrence coefficients necessary for the existence of a measure are computed to ensure the orthogonality of quasi-Geronimus Laguerre polynomial of order one. Numerical experiments concerning the zeros of the quasi-Geronimus Laguerre polynomial of order one are presented. Examples are provided to support further analysis.

Keywords: Orthogonal polynomials; Linear functional; Linear spectral transformations; Quasi-orthogonal polynomials; Recurrence relations
AMS Classification: 42C05.

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Thursday 27
16:00
Room C

Exceptional elliptic biorthogonal rational functions

Satoshi Tsujimoto

Abstract

Recently exceptional orthogonal polynomial sequence (XOPS) has been the subject of much research as one of the extensions of classical orthogonal polynomials. We consider exceptional type extensions for univariate bi-orthogonal rational functions by applying the Darboux transformation used to introduce XOPS to the case of generalized eigenvalue problems. This gives exceptional-type analogues of elliptic biorthogonal rational functions that appear in exact solutions of the elliptic Painlevé equation, etc., and investigates order jumps in rational function sequences. Moreover, we discuss biorthogonality relations among pairs of rational function sequences and elliptic hypergeometric equations in the exceptional type extensions.

Keywords: Elliptic hypergeometric series, Exceptional extensitons, Classical orthogonal polynomials
AMS Classification: 33C45, 33C47, 42C05.

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Thursday 27
12:30
Room B

Szegő Recurrence for Multiple Orthogonal Polynomials on the Unit Circle

Marcus Vaktnäs

Abstract

We generalize the direct and inverse Szegő recurrence relations to multiple orthogonal polynomials on the unit circle. This is also a direct analogue of the nearest neighbour recurrence relations [2], that generalize the three-term recurrence relation from orthogonal polynomials on the real line. From our recurrence relations we identify the generalized Verblunsky coefficients, find the partial difference equations satisfied by these coefficients, and show a Christoffel–Darboux formula. This is joint work with R. Kozhan, based on the paper [1].

Keywords: Multiple Orthogonal Polynomials, Orthogonal Polynomials on the Unit Circle, Verblunsky Coefficients, Christoffel–Darboux Formula
AMS Classification: 42C05, 47B36, 41A21.

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A Golub-Welsch version for simultaneous Gaussian quadrature

Tuesday 25
11:30
Room C

Walter Van Assche

Abstract

The zeros of type II multiple orthogonal polynomials can be used for quadrature formulas that approximate r integrals of the same function f with respect to r measures $\mu_1, \dots, \mu_r$ in the spirit of Gaussian quadrature. This was first suggested by Borges [1] in 1994, even though he does not mention multiple orthogonality. We give a method to compute the quadrature nodes and the quadrature weights which extends the Golub-Welsch approach using the eigenvalues and left and right eigenvectors of a banded Hessenberg matrix. This method was already described by Coussement and Van Assche [2] in 2005 but it seems to have gone unnoticed. We describe the result for $r = 2$ and give some examples.

Keywords: Multiple Orthogonal Polynomials, Gaussian quadrature, Banded Hessenberg Matrix
AMS Classification: 41A55, 65D32, 33C45, 41A21, 42C05

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Tuesday 25
16:00
Room C

Generalized derivatives and the construction of orthogonal polynomial sequences

Luis Verde-Star

Abstract

A linear operator P on the space of polynomials in one variable is a *generalized derivative* if there exists a polynomial sequence $\{u_k(t)\}_{k\geq 0}$ and a sequence of numbers c_k such that $Pu_k(t) = c_k u_{k-1}(t)$, for $k \geq 0$, where $c_0 = 0$ and c_k is nonzero for $k \geq 1$.

We will explain how certain generalized derivatives can be used to construct the hypergeometric and basic hypergeometric orthogonal polynomial sequences and also some polynomial sequences with several parameters that generalize the Hermite and Laguerre polynomials. We will also discuss the connection of generalized derivates with the monomiality principle and with differential operators of infinite order with polynomial coefficients. We work with the matrix representations of linear operators and use diagonal similarity and changes of bases in the space of polynomials. We use some of the results presented in [1].

Keywords: Orthogonal Polynomials, Generalized derivatives, Monomiality principle, Infinite matrices.
AMS Classification: 33C45, 44A45, 15A30.

References

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Meta Algebras and the Extension of the Askey Tableau to Biorthogonal Rational Functions

Satoshi Tsujimoto, Luc Vinet and Alexei Zhedanov

Monday 24
16:00
Room B

Abstract

Meta algebras with three generators are introduced. They subsume algebras of Askey-Wilson type and provide a unified algebraic description of hypergeometric orthogonal polynomials and companion biorthogonal rational functions that will be fully characterized. The latter special functions will be seen to arise as overlaps between eigenbases of generalized and ordinary eigenvalue problems on meta algebra modules. They will be shown to be bispectral. The framework extends the notion of Leonard pair. Certain examples will be discussed in details.

Keywords: Orthogonal Polynomials, Biorthogonal Rational Functions, Representation Theory.
AMS Classification: 33C47, 33C80, 33D45.

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Monday 24
16:30
Room C

Multiple orthogonal polynomials with hypergeometric moment generating functions

THOMAS WOLFS

ABSTRACT

I will discuss three families of multiple orthogonal polynomials associated with weights for which the moment generating functions are hypergeometric series with slightly varying parameters, see [9]. The weights are supported on the unit interval, the positive real line, or the unit circle and the multiple orthogonal polynomials can be seen as generalizations of the Jacobi, Laguerre or Bessel orthogonal polynomials. The type II Jacobi- and Laguerre-like multiple orthogonal polynomials appear naturally in random matrix theory: they arise as the average characteristic polynomial associated with the squared singular values of products of Ginibre and truncated unitary random matrices. I will explain how their zeros can be studied using the techniques from free probability developed recently in [4]. Particular cases of these polynomials have been investigated before in [6, 1, 8, 2, 3, 7]. The type I Bessel-like multiple orthogonal polynomials can be used to simultaneously approximate certain hypergeometric series and to provide an explicit proof of their $\mathbb{Q}$-linear independence. I will explain how these families can be extended to an even broader setting. For the Bessel-like family, this then leads to a generalization of Hermite's classical result on the transcendence of Euler's number e.

Keywords: multiple orthogonal polynomials, hypergeometric series, random matrices, Diophantine approximation.
AMS Classification: 33C45, 42C05, 60B20, 11J72.

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Monday 24
12:00
Room A

Orthogonal polynomials on domains of revolution

Yuan Xu

Abstract

We report recent results on orthogonal polynomials for a weight function defined over a domain of revolution, where the domain is formed from rotating a two-dimensional region and goes beyond the quadratic domains. Explicit constructions of orthogonal bases are provided for weight functions on a number of domains. Particular attention is paid to the setting when an orthogonal basis can be constructed explicitly in terms of known polynomials of either one or two variables. Several new families of orthogonal polynomials are derived, including a few families that are eigenfunctions of a spectral operator and their reproducing kernels satisfy an addition formula.

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L^2 Inequality for a Modified Struve Transform

Nesrin Yousfi

Monday 24
12:30
Room B

Abstract In this presentation, we aim to establish the analogue of the Heisenberg uncertainty principle for a generalized Fourier transform, called the modified Struve transform, denoted as S_α. We obtain:

$$\left(\int_0^{+\infty} x^2|f(x)|^2 d\mu_\alpha(x)\right)\left(\int_0^{+\infty} \lambda^2|S_\alpha f(\lambda)|^2 d\hat{\mu}_\alpha(\lambda)\right) \geq C_\alpha \left(\int_0^{+\infty} |f(x)|^2 d\mu_\alpha(x)\right)^2,$$

where C_α is a constant depending only α. The measures $d\mu_\alpha$ and $d\hat{\mu}_\alpha$ are weighted measures on $(0,+\infty)$ defined by $d\mu_\alpha(x) = x^{2\alpha+1}dx$ and $d\hat{\mu}_\alpha(x) = \frac{dx}{x^{2\alpha+1}}$.

The modified Struve transform is defined as:

$$S_\alpha f(x) = \int_0^{+\infty} (xt)^{1+\alpha}\mathbb{H}_\alpha(xt)f(t)dt,$$

where $\mathbf{H}_\alpha$ is the Struve function of order α. Note that the Struve function is a Bessel function of the second kind. Unlike Bessel function of first kind J_α, $\mathbf{H}_\alpha$ is the solution of the Bessel equation that is singular on 0. To invert S_α, we employ Titchmarsh's method, which involves applying the Mellin transform to invert asymmetrical Fourier transforms. Thus, we find the inverse transform, denoted S_α^{-1}, as:

$$S_\alpha^{-1}g(x) = \int_0^{+\infty} (xt)^{-\alpha}Y_\alpha(xt)g(t)dt,$$

where Y_α is the Bessel function of the second kind and order α.

Additionally, we prove an L^2 inequality for the modified Struve transform S_α, stated as follows:

Let $-2 < \alpha < 0$. For all f in $L^2(d\mu_\alpha)$, $S_\alpha f$ belongs to $L^2(d\hat{\mu}_\alpha)$ and satisfies:

$$\|S_\alpha f\|_{L^2(d\hat{\mu}_\alpha)} \leq C_1(\alpha)\|f\|_{L^2(d\mu_\alpha)},$$

where $C_1(\alpha)$ is a positive constant depending only on α.

Keywords: L^2 inequality, Modified Struve transform, inversion formula, Mellin transform, Bessel functions, Heisenberg Uncertainty principle.
AMS Classification: 42A38, 44A20, 26D10, 33C10.

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Wednesday 26
12:30
Room A

Umbra, extension and difference equation for q-Hermite polynomials

Umme Zainab

Abstract

In this paper, we established an umbra for 2-variable q-Hermite polynomials, followed by the introduction of 3-variable q-Hermite polynomials using the umbral approach. Employing umbral techniques, we delved into numerous intriguing properties associated with these polynomials. Additionally, we provided an extension, index duplication formula and argument duplication formula for q-Hermite polynomials.

Keywords: Umbra, q-Hermite polynomials, associated q-Hermite polynomials, extension of q-Hermite polynomials.
AMS Classification: 05A30, 05A40, 39A13.

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Tuesday 25
17:00
Room A

Bispectrality in different contexts and applications

Ignacio Zurrian

Abstract

In this talk we will discuss different situations of bispectral functions, over different rings and depending on variables of the same or different nature. We will also consider some applications and the role of the Darboux transformation.

Keywords: Orthogonal Polynomials, Approximation Theory, Applications, Matrix-valued special functions
AMS Classification: 37K35, 16S32, 39A70.

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Posters

Monday 24
Tuesday 25
17:30
Poster Session

A note on Laguerre truncated polynomials and quadrature formula

Juan C. García-Ardila and Francisco Marcellán

Abstract

In this communication we show some results concerning to Gaussian quadrature rules based on orthogonal polynomials associated with a weight function $w(x) = x^{\alpha}e^{-x}, \alpha > -1$, supported on an interval $(0, z)$, $z > 0$. The modified Chebyshev algorithm is used in order to test the accuracy in the computation of the coefficients of the three term recurrence relation, the zeros and weights as well as the dependence on the parameter z.

Keywords: Quadrature formulas, Laguerre truncated polynomials, Modified Chebyschev algorithm.
AMS Classification: 42C05, 33C50.

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Bernstein-type Operators based on the Jacobi inner product

David Lara Velasco and Teresa E. Pérez

Monday 24
Tuesday 25
17:30
Poster Session

Abstract

Bernstein polynomials were introduced by S. Bernstein in 1912 to provide a constructive proof of the Weierstrass approximation theorem. In this way it was established that every continuous function defined in the interval $[0, 1]$ can be uniformly approximated by Bernstein polynomials in such interval.

In this work we study a modification of the Bernstein operator that was studied in [2] by means of the Jacobi inner product. We analyze its properties on different types of functions and their possible applications.

Keywords: Orthogonal Polynomials, Approximation Theory, Applications
AMS Classification: 33CXX, 42CXX.

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Monday 24
Tuesday 25
17:30
Poster Session

Eigenvalue Rigidity of the Jacobi Unitary Ensembles

Dan Dai, Chenhao Lu

Abstract

This project aims to establish the eigenvalue rigidity of Jacobi unitary ensembles. We want to find an optimal bound for the fluctuations of eigenvalues away from their limiting values. Different from the Gaussian unitary ensemble, hard edges appear in the Jacobi case, hence the eigenvalues are expected to concentrate near the edges.

The main idea of our proof is to combine the extreme value theory of log-correlated Gaussian field, especially the fractal properties of the Gaussian multiplicative chaos measure, together with asymptotic analysis of the Hankel determinants with Fisher-Hartwig singularities, where Riemann-Hilbert approach is adopted. Some estimates of the exponential moments of an asymptotically Gaussian process are also obtained.

Keywords: Eigenvalue rigidity; Jacobi Unitary ensemble; Gaussian multiplicative chaos; Exponential moments; Hard edge; Riemann-Hilbert problems.
AMS Classification: 60B20, 60G57, 33C45, 35Q15.

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Monday 24
Tuesday 25
17:30
Poster Session

Symbolic computation of the coefficients of the second–order difference equation for general Sobolev–type orthogonal polynomials

Galina Filipuk, Juan F. Mañas–Mañas and Juan J. Moreno–Balcázar

Abstract

We consider the monic ortogonal polynomials, $Q_n(x)$, with respect to the general Sobolev–type inner product

$$(f,g)_S = \int f(x)g(x)\varrho(x)dx + M\mathscr{D}_{q,\omega}^{(j)} f(c)\mathscr{D}_{q,\omega}^{(j)} g(c),$$

where $\varrho(x)$ is a weight function with support on the real line, $c \in \mathbb{R}$, $M > 0$, j is a nonnegative integer and $\mathscr{D}_{q,\omega}$ is the operator introduced by Hahn defined by

$$\mathscr{D}_{q,\omega} f(x) = \begin{cases} \dfrac{f(qx+\omega)-f(x)}{(q-1)x+\omega}, & \text{si } x \neq \omega_0; \\ f'(\omega_0), & \text{si } x = \omega_0, \end{cases}$$

with $0 < q < 1$, $\omega \geq 0$ and $\omega_0 = \frac{\omega}{1-q}$.

In [1], we proof that the polynomials $Q_n(x)$ satisfy the following second–order difference equation,

$$\sigma_{1,c,n}(x)\mathscr{D}_{q,\omega}^{(2)} Q_n(x) + \sigma_{2,c,n}(x)\mathscr{D}_{q,\omega} Q_n(x) + \sigma_{3,c,n}(x)Q_n(x) = 0, \quad n \geq 2,$$

where $\sigma_{1,c,n}(x)$, $\sigma_{2,c,n}(x)$ and $\sigma_{3,c,n}(x)$ are explicitly known functions. Now, we present a program based on Mathematica® that symbolically calculates the coefficients $\sigma_{1,c,n}(x)$, $\sigma_{2,c,n}(x)$ and $\sigma_{3,c,n}(x)$ (see [2]).

The corresponding code is freely available at `https://w3.ual.es/GruposInv/Tapo/SODE.nb`.

This is a joint work with Galina Filipuk and Juan J. Moreno–Balcázar.

Keywords: Sobolev Orthogonal Polynomials, Second–Order Difference Equation, Symbolic Computation
AMS Classification: 33C47, 42C05, 34A05.

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Monday 24
Tuesday 25
17:30
Poster Session

Bessel Function Solutions of the Fifth Painlevé Equation

Ben Mitchell

Abstract

We explore rational solutions of the fifth Painlevé equation. This equation exhibits special function solutions for specific parameter values, expressed in terms of Kummer functions. It is well-known that the third Painlevé equation has special function solutions represented by Bessel functions. By utilizing connection formulae between Kummer functions and modified Bessel functions, we demonstrate that the fifth Painlevé equation also admits Bessel function solutions. Furthermore, we investigate the structure of the roots of these solutions.

Keywords: Painlevé equations, Kummer functions, Bessel functions

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EPSRC

Asymptotic Zero Behavior of some Meijer G–Functions

Cristina Rodríguez–Perales, Juan F. Mañas–Mañas and Juan J. Moreno–Balcázar

Monday 24
Tuesday 25
17:30
Poster Session

Abstract

In this contribution our aim is to study the asymptotic behaviour of the zeros of some cases of the Meijer G–function. In particular, we focus our attention on the functions $G^{1,n}_{p,q}\left(z;\ \begin{matrix}\alpha_1,\dots,\alpha_p\\ \beta_1,\dots,\beta_q\end{matrix}\right)$ and $G^{m,1}_{p,q}\left(z;\ \begin{matrix}\alpha_1,\dots,\alpha_p\\ \beta_1,\dots,\beta_q\end{matrix}\right)$, subject to certain conditions on the parameters. With that objective, we tackle the study of the Mehler–Heine asymptotics for these G–functions. Finally, we show some numerical experiments which illustrate the obtained convergence results.

Keywords: Special functions, Meijer G–functions, Asymptotics, Zeros
AMS Classification: 33C47, 33C60.

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A new approach to zeros of multivariate orthogonal polynomials

Monday 24
Tuesday 25
17:30
Poster Session

José L. Ruiz Benito and Teresa E. Pérez

Abstract

The zeros of an orthogonal polynomial in one variable with respect to a positive definite linear functional are a finite set of points on the real line. For each degree, this orthogonal polynomial is unique except for a multiplicative constant, so its set of zeros is actually a property of the functional.

For an orthogonal polynomial of several variables, traditionally its zeros have been considered as the algebraic curve on which the polynomial becomes zero. Thus, when expressing orthogonal polynomials of a certain degree in vector form, it is not always possible to find common zeros, and when obtaining another vector of orthogonal polynomials by multiplying by an invertible matrix, the zeros are totally different.

In this work, we introduce a new definition of zero for multivariate polynomials, which generalizes the definition of zero in one variable and through which the set of zeros of a vector of orthogonal polynomials is an intrinsic property of the functional. In addition, we analyze some properties in relevant families.

Keywords: Orthogonal Polynomials, Multivariate Orthogonal Polynomials, Zeros of Orthogonal Polynomials
AMS Classification: 33C50, 42C05.

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Generalization of the Multiple Orthogonality to the Bivariate Case

Monday 24
Tuesday 25
17:30
Poster Session

J. Antonio Villegas, Lidia Fernández

Abstract

Polynomials known as Multiple Orthogonal Polynomials (MOPs) in a single variable are polynomials that satisfy orthogonality conditions concerning multiple measures and play significant role in several applications such as Hermite-Padé approximation, random matrix theory or integrable systems. However, this theory has only been studied in the univariate case. In this poster, some generalized definitions of the two main types of multiple orthogonality are given, together with some examples and extended results.

Keywords: Orthogonal Polynomials, Approximation Theory, Applications, Multiple orthogonality.
AMS Classification: 33C45, 33C50, 42C05.

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Monday 24
Tuesday 25
17:30
Poster Session

On Laurent biorthogonal polynomials and Painlevé-type equations

Xiao-Lu Yue, Xiang-Ke Chang and Xing-Biao Hu

Abstract

In this paper, we investigate Laurent biorthogonal polynomials with a weight function of three parameters, i.e. $z^{\alpha}e^{-t_1 z-\frac{t_2}{z}}, z \in (0,+\infty)$, $(t_1 > 0,\ t_2 > 0,\ \alpha \in \mathbb{R})$. First, the structure relation of the Laurent biorthogonal polynomials is found with the aid of biorthogonality. Then we derive an alternate discrete Painlevé II by considering the compatibility condition of the three-term recurrence relation and the structure relation. In addition, we make use of the relativistic Toda chains and nonlinear difference system to obtain two continuous Painlevé-type differential equations.

Keywords: Laurent biorthogonal polynomials; three-term recurrence relation; structure relation; compatibility condition; Painlevé equations

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